

Example 12

Write $\frac{420}{66}$ in lowest terms.

Solution:

$$420 = 6 \cdot 66 + 24 \text{ (finding gcd)}$$

$$66 = 3 \cdot 24 - 6 \text{ (remember we may use negative numbers when convenient)}$$

$$24 = 4 \cdot 6 + 0$$

$$\text{gcd}(420, 66) = 6$$

$$420/6 = 70 \text{ (divide numerator by gcd)}$$

$$66/6 = 11 \text{ (divide denominator by gcd)}$$

$$\frac{420}{66} = \frac{70}{11} \text{ (} \frac{70}{11} \text{ is in lowest terms)}$$

This discussion does not constitute a proof, but the truth of the following theorem should be clear.

Theorem

Let $\frac{c}{d}$ be a rational number. Then there is a rational number $\frac{a}{b}$ in lowest terms such that $\frac{c}{d} = \frac{a}{b}$.

This theorem will play an important role in the proofs of the next section.

Exercises 1.2

For exercises 1–4, use the method of examples 3 and 4 to choose from among

$<$, $=$ and $>$ to fill in the blank.

1. $\frac{29}{11}$ _____ $\frac{137}{50}$ 2. $\frac{3}{7}$ _____ $\frac{8}{19}$

3. $\frac{130}{57}$ _____ $\frac{11570}{5073}$ 4. $\frac{48}{441}$ _____ $\frac{311}{2959}$

For exercises 5–10, use the definitions to perform the desired operation.

5. $\frac{1}{2} + \frac{4}{7}$ 6. $\frac{3}{5} + \frac{9}{4}$

7. $\frac{4121}{8003} + \frac{805}{7192}$ 8. $\frac{62}{3025} + \frac{11}{27}$

9. $\frac{14}{17} \cdot \frac{3}{2}$ 10. $\frac{4}{7} \cdot \frac{9}{13}$

For exercises 11–14, write the rational number in decimal form. Do not use approximations.

11. $\frac{127}{80}$ 12. $\frac{14}{250}$

13. $\frac{15}{11}$ 14. $\frac{24}{91}$