Transposition errors are caught in a similar manner. A *transposition* of two digits occurs when the order of the two digits is reversed; for example, reading the ISBN of example 2 as 0–201–25983–1 is a transposition of d_5 and d_6. That would change c by changing $6 \cdot 5 + 5 \cdot 2$ to $6 \cdot 2 + 5 \cdot 5$. The change in c is then 6(–3) + 5(3), since the 5 was reduced to 2 and the 2 was increased to 5. But 6(–3)+5(3) = –3, $c \equiv 8 \pmod{11}$, and the error is caught. The change in c is the distance between the positions of the transposed digits times the difference between the transposed digits, each of which must be less than 11. Since 11 is prime, the change is not a multiple of 11 and the error is caught.

There you have it—prime numbers and modular arithmetic put to good use, improving the efficiency of commerce! A code such as ISBN–10 that always detects certain types of errors is called an *error-detecting code*. If you think that's neat, then get this—there are even *error-**correcting** codes* (see inset)!

UPC

The Universal Product Code (UPC) is placed on nearly everything that is sold in retail stores. The UPC symbol contains the UPC in both barcode and numeric form. The calculation of the UPC check digit and verification of a UPC are very similar to the ISBN–10.