

Mathematical Lives

BENOIT
MANDELBROT

Reshaping the World

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Prologue

Clouds Are Not Spheres

As summer began in 1982, *Star Trek* fans were flocking to movie theaters for *The Wrath of Khan*, the second feature film reuniting Captain Kirk, Spock, and the crew of the original starship *Enterprise*. More than a decade after the original TV series had been canceled, audiences thrilled to witness the struggle between the *Enterprise* and its old enemy, Kahn Noonien Singh. What almost nobody knew at the time, however, is that they were also watching cinematic and mathematical history.

Toward the middle of the film, Kirk, Spock, and Dr. McCoy watch a classified Starfleet presentation describing “the Genesis Device,” a new invention for turning dead planets into ones that can support life. As the inventor describes the process, a computer animation sequence shows the device in action. For a full minute, the characters and the audience watch as a barren, cratered planet is transformed into a new world with oceans, mountains, clouds, and plants.

The Genesis Device may have been a fictional invention, but real inventions brought it to life. That sequence was the first fully-computerized animation ever used in a feature

film. Its creator, Loren Carpenter, who went on to become co-founder and chief scientist of Pixar Animation, made it using a revolutionary new field of mathematics called *fractal geometry*.

Computer graphics have advanced far beyond those used in *The Wrath of Khan*, but they still rely on fractal geometry for their realistic look. And over the decades, fractal geometry has been used for much more than making movies and video games. It's the reason that cars and mobile phones no longer have long antennas attached to them. It's used to model structures in nature, from the blood vessels in our bodies to the drainage pattern of a landscape. And it's used to study complex and chaotic systems like the weather and the stock market.

But what is fractal geometry? Essentially, it's a mathematical way of representing the "roughness" found in nature. For hundreds of years, geometry was based on the *Elements*, written by the ancient Greek mathematician Euclid, but Euclid's geometry falls short when trying to describe something like a cloud, a mountain, a coastline, or a tree. In 1983, the groundbreaking book *The Fractal Geometry of Nature* declared, "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

That book was written by the mathematician who discovered fractal geometry, Benoit Mandelbrot (pronounced "Ben-WAH MAN-del-brawt"), a fellow at IBM's T.J. Watson Research Center and a visiting professor at Harvard University. Mandelbrot had introduced the word *fractal*

in 1975, after observing the same patterns in seemingly unrelated fields, such as those found in telephone signals, financial trends, and coastline measurements. The more he looked, the more he saw a property called *self-similarity*, which is at the root of fractal geometry. An object is said to be self-similar when pieces of it look just like the whole. (Think of a head of broccoli. Break off a stalk—what does it look like?) Mandelbrot’s exploration of how and why that happened produced the most revolutionary development in geometry since the seventeenth century.

Finding connections among such different subjects required someone whose life had gone from place to place and whose career had gone from one field to another. Mandelbrot had certainly done both. “Very often when I listen to the list of my previous jobs,” he once remarked, “I wonder if I exist. The intersection of such sets is surely empty.” This is the story of how that life led him to one of the most exciting new fields in mathematics today.

Chapter One

Uncle Szolem

When Benoit Mandelbrot was born, on November 20, 1924, the country of his birth, Poland, wasn't much older than he was. There had been a Polish nation since the Middle Ages, but in the late 1700s it was conquered and divided up by the major powers around it. The capital, Warsaw, had belonged to the Russian Empire until 1915, when it was captured by Germany during World War I. When Germany was defeated, the Poles declared themselves an independent country again, a status later confirmed by the Treaty of Versailles.

Benoit's father Karl had been born in Warsaw, but the family had deep roots in the Jewish community around Vilnius, which today is the capital of Lithuania but has been united with Poland in one way or another for centuries. His mother Lurie's family also came from the Vilnius area; Lurie herself was born in a town near there. She and Karl met as schoolchildren in Warsaw, where Karl was a classmate of Lurie's older brother. By the time the German army marched into Warsaw, they were married and both trying to establish themselves in their professions. They fled eastward, arriving

in the city of Kharkov (now part of Ukraine and called Kharkiv), but any peace they found there was short-lived. Soon the Russian Revolution and civil war broke out, forcing them to flee again. This time they went south to Sevastopol in Crimea, then west to the Romanian city of Constanța, before finally returning to Warsaw in 1919.

The age in which they lived made it hard for Benoit's parents to establish themselves, as one upheaval after another forced them to start over again. Karl had gone to trade school and had learned bookkeeping, but circumstances took him into the clothing industry, where he set up and ran a series of small businesses. His passion, however, was understanding how machines worked. In the 1930s, he tried to become a freelance inventor, even earning a patent for one of his creations, but the Great Depression forced him back into steadier work.

Lurie, meanwhile, was a dentist. Her grandfather, expressing an unusually progressive attitude for the time, had insisted that all of his granddaughters become doctors, and Lurie chose dentistry because she thought it would interfere less with motherhood. The Depression affected her career as well, as most patients waited until they were in unbearable pain before coming to her. Benoit later recalled one patient, though, who came for some more extensive work and paid well for it. He was a man whose girlfriend wouldn't kiss him because of the bad breath his rotten teeth caused, and he wanted those teeth fixed. Unfortunately, he worked in a slaughterhouse, and the Mandelbrot home had to be aired out after each visit to get rid of the manure smell.

Benoit was not the Mandelbrots' first child. An older brother had been born some years before but had died in a meningitis epidemic. The loss devastated his mother, who never stopped grieving for him. However, Benoit's birth lifted her spirits, as did the birth of his younger brother Léon fifteen months later, but losing one son affected the way she raised the other two.



Benoit and Léon on the streets of Warsaw with their mother (left, holding Benoit's hand) and their Aunt Raya

For one thing, fear of another epidemic kept Lurie from sending her sons to school. Benoit's education began with lessons from his Uncle Loterman instead. He learned how to read and write quickly, but he ran into trouble with other subjects. Uncle Loterman was a "chronically unemployed intellectual" with no teaching experience and poor organizational skills. He hated learning by memorization, so he never taught Benoit some basic lessons like the multiplication tables—something Benoit claimed to struggle

with throughout his life. On the other hand, he did teach Benoit to speed-read and to play chess, and he filled their days with stories from Poland's past and discussions about current events, both enhanced by the large collection of maps that filled his home. It was an unusual education, but it inspired in Benoit the wide-ranging curiosity that drove his discoveries throughout his career.

When Benoit finally went to public school in the third grade, he was entering a school system that was hard at work helping Poland establish a national identity. It wasn't an easy task. After centuries under the rule of one foreign empire or another, more than a quarter of the country's people were not ethnically Polish. Ukrainians made up the largest percentage, followed by Jews, Belarusians, and Germans. Officially, the government tried to impose the idea that Poland was a "happy multinational country where all the ethnic problems of the past have been solved." In reality, however, many Poles regarded non-Poles as foreigners—even ones born in Warsaw, like the Mandelbrot children—and wanted them to leave.

Of the four ethnic groups, Jews often faced the most discrimination. They were the only non-Christian group and the only one without a nearby homeland to support them. Schools were segregated between Jews and Christians, and limits were placed on the number of Jews who could go on to higher education or hold skilled professional jobs. The situation improved somewhat under the rule of Polish leader Józef Piłsudski, but life was still hard. Luckily for Benoit, his family was able to shelter him from the worst of the

country's anti-Semitism, and he spent four uneventful years in the classroom of a teacher named Mrs. Goldszlakowa, who accompanied his class from one grade to the next.

Help came to the family from another of Benoit's uncles, Szolem Mandelbrot (sometimes spelled "Mandelbrojt"). Karl's younger brother by sixteen years, Szolem was the first family member to attend an academic high school instead of a trade school or religious school and the first to go on to a university that wasn't medical school. Studying in Kharkov, he attended lectures by a Russian mathematician named Sergei Bernstein, who had just finished his Ph.D. in Paris. Bernstein introduced Szolem to the work of French mathematicians like Henri Poincaré, which captured Szolem's imagination and held it for the rest of his career. He moved to Paris, where he earned a doctorate at the prestigious Paris-Sorbonne University in 1923. A year later he was awarded a fellowship to continue his studies in the United States, and in 1926 he became an assistant professor at the Rice Institute (now Rice University) in Houston, Texas.



Szolem Mandelbrot (in the center, wearing the dark suit) with other members of the Rice Institute Mathematics Department, 1927

Szolem’s specialty was *mathematical analysis*, a field that developed out of calculus that involves studying theories related to an elementary concept in calculus called a *limit*. His doctoral thesis was about a special set of equations called *Taylor series*, which have this form:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n$$

(Don’t worry if you don’t recognize all of those symbols. We won’t go into the details here.)

Szolem was interested in “pure” mathematics—exploring or creating mathematical concepts that couldn’t be applied to another field, such as science or engineering or finance. According to an acquaintance, Szolem said that mathematics “are the same thing as poetry, that one invents mathematical beauty, and that true mathematicians never do arithmetic.” For Benoit, having such a mathematically talented uncle inspired him to pursue mathematics himself. But as he found his own specialties and fields of study, he and Szolem grew apart professionally. Benoit’s studies all involved mathematical applications and expressing mathematical ideas through pictures. For Szolem, that kind of mathematics wasn’t “pure” enough.

After a year at the Rice Institute, Szolem attained French citizenship and returned to France. Two years later he became a full professor at the University of Clermont-Ferrand (birthplace of another great mathematician, Blaise Pascal, coincidentally). He kept close ties with colleagues

in Paris, befriending a number of French intellectuals, and in 1934 he joined several other mathematicians to form a group that published under the pseudonym “Nicolas Bourbaki.” Their goal was to create a set of textbooks covering all of the subjects in pure mathematics. By 1938, Szolem had moved to Paris full-time, accepting a new position at the Collège de France as the Chair of Analytical Mechanics and Celestial Mechanics.

Szolem’s assimilation into French society produced another way in which he helped to shape Benoit’s life. As the grip of the Great Depression and the danger of anti-Semitism both grew steadily in Poland, Szolem began suggesting that Karl, Lurie, and their sons join him in Paris. The French economy was suffering too, but even so, there were more opportunities there than in Warsaw, and Benoit’s family could draw on Szolem’s resources and connections for help. In 1931 Karl accepted the offer, but only for himself. He went to Paris, while Lurie remained in Warsaw with Benoit and Léon, waiting to see if Karl could get his business reestablished.

The family continued that way for five years, until the time came for Benoit to apply to high school in 1936. A year earlier, his cousin Mirka had placed first on the entrance exam for the best girls’ high school in Warsaw, but she was turned down for admission because a more well-connected family took the one place available for a Jewish student. Szolem had to reach out to his academic contacts, going all the way to Poland’s most prominent mathematician, Waclaw Sierpiński, before Mirka was admitted. (Remember

Sierpiński's name, by the way. We'll hear more about him later.) There were even fewer high schools available to a Jewish boy, meaning that Benoit's chances of success at finding a school were even slimmer than Mirka's had been. At the same time, the death of Józef Piłsudski in 1935 brought in a new government that enacted a number of anti-Semitic policies. It looked increasingly like a move to France was the family's only option.

Even so, the decision was a hard one. Karl was still struggling to establish himself in business. He wasn't certain that he could earn enough money to support the family. For Lurie, things would be even harder. Not only did she have to leave her family and friends, but she also had to give up her dental practice. She would have to become a housewife living in one of Paris's poorer neighborhoods. However, both Karl and Lurie considered the sacrifice worth making because of the opportunities it would give their sons.

The time for the move approached, but not fast enough for the Mandelbrots' landlord, who evicted them from their apartment as soon as the lease ran out so he could remodel it for his son. The family had to stay with Lurie's sister Raya for a time before they could leave. Finally, with neighbors, friends, and Lurie's former patients coming to see them off, they boarded a chartered train for refugees and left Warsaw. The train was old and slow and frequently stopped to let faster trains go by along its 850-mile trip from Warsaw to Paris. In an ominous sign of things to come, when the train crossed through Nazi Germany, the doors were padlocked to keep anyone from going in or out.

Karl was waiting for them when they arrived, along with his sister Fanny, who also lived in Paris. It was time for the family's new life to begin. But it wouldn't be long before that life was completely upended, and the entire family would find itself in mortal danger.