Recall that the natural numbers are closed under addition and multiplication. Allowing all subtractions of natural numbers gave us the integers, while throwing in all divisions of integers (except by 0) yielded the rationals. It turns out, though, that including all roots of positive rationals and all odd roots of negative rationals is still not enough to produce all the algebraic numbers, much less the reals. This was first shown by

A Most Fascinating Number

Perhaps no number is more celebrated than $\pi = 3.14159\ldots$, the ratio of a circle's diameter to its circumference. There are pi t-shirts, pi songs, pi posters containing a million digits of pi, pi websites, pi poetry, books about pi, even a Pi Day (March 14, i.e., 3–14). While some of the aforementioned items are a bit overboard, π certainly has many uses in mathematics and thus merits continued study.

One of the more interesting properties of π is that its digits seem to be random. By the time Johann Lambert proved in 1761 that π is irrational, over a hundred decimal digits of π were known. Lambert's work was sufficient to show that the decimal representation of π never terminates nor repeats, but irrationality does not guarantee the absence of a pattern—for instance, the number $.101001000100001\ldots$ never terminates nor repeats but still exhibits an easily discernible pattern. Nor does Ferdinand von Lindemann's 1882 proof that π is transcendental (not algebraic) help in this regard. Thus mathematicians have continued to examine the decimal expansion of π, and with the help of computers have now calculated over a trillion digits. Although these digits have passed every statistical test for randomness so far, no one has been able to prove that π is *normal*, that is, that every digit occurs equally often, every two-digit block occurs equally often, etc. In addition to practical uses, the lack of a known pattern makes it a natural for testing memorization abilities; believe it or not, on October 3, 2006 in Kisarazu, Japan, Akira Haraguchi is reported to have recited π to 100,000 decimal places!

Pi has also been used by some to dispute the accuracy of the Bible. 1 Kings 7:23–26 describes a large bronze basin constructed for the temple as measuring 10 cubits (180 inches) from one brim to the other and 30 cubits (540 inches) in circumference. This would seemingly give π a fairly inaccurate value of 3. But the passage also states that the thickness of the object was a handbreadth, or 1/6 cubit (3 inches). If the diameter is an outer diameter and the circumference an inner circumference, then the inner diameter would be 174 inches, giving a much closer ratio of $\frac{540}{174} = 3.103$ (see www.history.mcs.st-andrews.ac.uk/Extras/Graf_theory.html). There is therefore no reason to doubt the given measurements to the number of significant digits used.