Example 12

Write $\frac{420}{66}$ in lowest terms.

Solution:

$420 = 6 \cdot 66 + 24$ (finding gcd)
$66 = 3{\cdot}24 - 6$ (remember we may use negative numbers when convenient)
$24 = 4 \cdot 6 + 0$
$\gcd(420, 66) = 6$
$420/6 = 70$ (divide numerator by gcd)
$66/6 = 11$ (divide denominator by gcd)
$\frac{420}{66} = \frac{70}{11}$ ($\frac{70}{11}$ is in lowest terms)

This discussion does not constitute a proof, but the truth of the following theorem should be clear.

Theorem

Let $\frac{c}{d}$ be a rational number. Then there is a rational number $\frac{a}{b}$ in lowest terms such that $\frac{c}{d} = \frac{a}{b}$.

This theorem will play an important role in the proofs of the next section.

Exercises 1.2

For exercises 1–4, use the method of examples 3 and 4 to choose from among

<, = and > to fill in the blank.

1. $\frac{29}{11}$ ______ $\frac{137}{50}$
2. $\frac{3}{7}$ ______ $\frac{8}{19}$
3. $\frac{130}{57}$ ______ $\frac{11570}{5073}$
4. $\frac{48}{441}$ ______ $\frac{311}{2959}$

For exercises 5–10, use the definitions to perform the desired operation.

5. $\frac{1}{2} + \frac{4}{7}$
6. $\frac{3}{5} + \frac{9}{4}$
7. $\frac{4121}{8003} + \frac{805}{7192}$
8. $\frac{62}{3025} + \frac{11}{27}$
9. $\frac{14}{17} \cdot \frac{3}{2}$
10. $\frac{4}{7} \cdot \frac{9}{13}$

For exercises 11–14, write the rational number in decimal form. Do not use approximations.

11. $\frac{127}{80}$
12. $\frac{14}{250}$
13. $\frac{15}{11}$
14. $\frac{24}{91}$