Transposition errors are caught in a similar manner. A *transposition* of two digits occurs when the order of the two digits is reversed; for example, reading the ISBN of example 2 as 0–201–25983–1 is a transposition of d_5 and d_6. That would change c by changing $6 \cdot 5 + 5 \cdot 2$ to $6 \cdot 2 + 5 \cdot 5$. The change in c is then $6(-3) + 5(3)$, since the 5 was reduced to 2 and the 2 was increased to 5. But $6(-3)+5(3) = -3$, $c \equiv 8 \pmod{11}$, and the error is caught. The change in c is the distance between the positions of the transposed digits times the difference between the transposed digits, each of which must be less than 11. Since 11 is prime, the change is not a multiple of 11 and the error is caught.

There you have it—prime numbers and modular arithmetic put to good use, improving the efficiency of commerce! A code such as ISBN–10 that always detects certain types of errors is called an *error-detecting code*. If you think that's neat, then get this—there are even *error-**correcting** codes* (see inset)!

Digital Signals and Noise Reduction

Error-correcting codes work by using several cleverly-designed check digits. First, errors are detected by making calculations on the given number to see if the required value(s) is obtained, in a manner similar to the codes in this section. The codes are designed so that when certain types of errors occur, no two different errors of that type could give the same result from the calculations. Knowing the result therefore means we know which error occurred, and it can then be corrected!

The ability to correct errors is what makes digital signal transmission so much better than analog. When transmitting or reading signals, whether radio or television waves through the air, satellite signals through space and the atmosphere, or telephone or cable television through wires, the signal may be corrupted. It is this "interference" that once upon a time caused fuzzy television pictures and quite a bit of noise in long-distance telephone calls. With modern computing technology, such signals can be encoded digitally before transmission. During transmission corruption still occurs, but with the use of error-correcting codes the message can be mostly repaired upon receipt, resulting in much clearer pictures, better sound quality, and greater success in transferring data. Practically all of our modern electronic communication and entertainment devices—including those you may have already used today—rely upon error-correcting codes.

UPC

The Universal Product Code (UPC) is placed on nearly everything that is sold in retail stores. The UPC symbol contains the UPC in both barcode and numeric form. The calculation of the UPC check digit and verification of a UPC are very similar to the ISBN–10.