Problemoids

Grade 5
Math Mentor

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Pascal called this triangle the Arithmetic Triangle. He did not design the triangle—there is evidence of its use by Omar Khayyam in the twelfth century—but it bears his name because he discovered many of its properties.

The cover of *Problemoids* illustrates one of the properties. Using two colors, one for the even numbers and one for the odd numbers, interesting triangular patterns appear. Notice the white triangles running through the center of Pascal's Triangle. The top "triangle" has one white dot, the second has six white dots, and the third has twenty-eight white dots. Two of these numbers, 6 and 28, are "perfect numbers"; that is, each is the sum of its divisors (excluding itself): 6 = 1 + 2 + 3 and 28 = 1 + 2 + 4 + 7 + 14. You might want to draw an even larger version of Pascal's Triangle than the one on the cover.

The rows and diagonals of Pascal's Triangle have surprising and useful properties, some of which can help you solve several problems in the *Problemoids* program. For example, suppose you add the numbers in any diagonal down to some point on that diagonal. There is an easy way to find the sum without performing the actual addition. Add the numbers on the third diagonal down to 45. Notice that 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 = 165 and 165 is the number on the fourth diagonal just below 45. Try several other diagonal sums. You'll find the same thing happens every time.

You may discover a few other properties as you solve the problems below:

1. What is the sum of the numbers in the tenth row of Pascal's Triangle? (It is customary to call the top row of Pascal's Triangle the 0th row. The tenth row begins: 1, 10, 45, ...)
2. How many numbers in the thirteenth row are divisible by 13?
3. How many odd numbers are there in the thirty-first row?

The answers to these problems appear on page 8.

NO PEEKING!

Implementation

The *Problemoids* program does not demand significant teacher involvement, although teachers may choose to participate quite actively. In either case the following recommendations will suffice for management of the programs.

1. ASSIGN THE PROBLEMS IN SEQUENCE.

Until the teacher becomes familiar with the problems, hints, answers, and solutions in *Problemoids,* we recommend assigning problems according to the order in which they appear in the student problem book. Generally, the position of the problem in sequence reflects its level of difficulty; that is, less difficult problems appear earlier and more difficult problems appear later in the sequence. Also, students' understanding of a problem appearing later in the sequence may be contingent on their familiarity with an earlier problem.

2. DIRECT STUDENTS TO MAINTAIN A RECORD OF THEIR WORK.

Students should maintain a record of their work toward solutions of problems. Space for work on each problem has been provided in the student book to assist students in organizing and keeping up with their solutions. The solution of a problem may be attempted over several sittings. For this reason, students need one fixed workspace to help them organize the solution and recall what they attempted in previous work.

If students do not work in their problem books, they should maintain their work in individual notebooks. A record that shows how individual problems were solved and how various strategies were employed may be beneficial for solving future problems.

3. DIRECT STUDENTS TO THE HINTS SECTION OF THE BOOK AND ENCOURAGE MODERATE USE OF THE HINTS.

If students feel blocked in their solution of a problem, they should be encouraged to use one or more hints.

Hints to a given problem are not grouped together, but appear separately to encourage using hints one at a time. For your convenience, the teacher's manual has the hints grouped together. Students probably will obtain the most benefit from a hint by allowing themselves time for reflection instead of reading through all of the hints to a problem quickly. Since this factor is difficult to control, the hints have been designed to provide instructional value in either case.

4. ALLOW STUDENTS AMPLE TIME TO THINK ABOUT AND SOLVE EACH PROBLEM.

The authors suggest assigning one or two problems at the beginning of a week. Since many of the problems are quite challenging, students usually will not be able to solve an individual problem during one class period.

Students will need to spend time understanding a problem, investigating possible approaches to its solution, reflecting upon how to utilize hints, debugging unsuccessful

Appendix III: Classification of Problems According to Mathematical Topic

● = primary classification
★ = secondary classification

	Sets	Number Numeration	Operations	Algebra	Geometry	Measurement	Probability & Statistics
1. Yin and Yang	●						
2. Inflation		●	★				
3. Dribbles		●			★		
4. A New Game Plan			★	●			
5. What's the Difference		★	●				
6. Where There's a Will, There's a Way					●	★	
7. Bit Buy Byte			★	●			
8. Confronting the Clock		●				★	
9. Candydextrous			★	★		●	
10. Aw Sum Gauss		●	★				
11. The Muse Family	●						
12. A Ball with the Bearings			●	★		★	
13. Sexagenarian				●			
14. The Toothpick Fairy					●		
15. Lots of Dots		●					★
16. A Liter Expedition			★			●	
17. Splitting the Spoils		★					●
18. Sweet Enigma			★	●			
19. Crack the Cube					●		
20. Fifty-Point Question		●			★		★
21. A Little More			●	★			
22. Check 'er Balance				●			
23. Caveat Emptor			★		★	●	
24. M&M's		●					
25. Hoppin' to Win			●				

	Sets	Number Numeration	Operations	Algebra	Geometry	Measurement	Probability & Statistics
26. The Case of the Quick Bookcases		★	★	●			
27. Hex Symbol					●		
28. Pause for Claws			★	●			
29. The Wandering Bishop		★					●
30. Computing the Odds		●					
31. Distributing the Wealth			●	★			
32. Friendly Finish				●		★	
33. Getting Even	●		★				
34. An Almost Perfect Place		●					★
35. Chew on This One					★	●	
36. A Walking Conundrum				●		★	
37. Hundreds Censored!	★		●				
38. Try Your Luck					●		
39. Quick Candlestick				★		●	
40. Partly Popular			★	●			
41. An Odd Gift		●					
42. Fair Ball?							●
43. Creating Cranapple			★			●	
44. Relatively Unknown		★	●				
45. Name Dropping		●					★
46. Parsimonious Purchaser	●						
47. Installation Free					●		
48. Quintillion Plus		●	★				
49. Three-Ouarter Dime Jingle				●			★
50. No Heads Is Better Than One							●

Meg and Charlie usually order one mushroom and sausage pizza at C & E Pizzeria. This pizza has a diameter of 50 centimeters and costs $5.00. Last week, however, C & E had a special: three 25-centimeter pizzas for $5.00.

"Let's try the special," Charlie said, expecting to gorge himself with the wonderfully oily morsels of the C & E specialty. "This place will go broke if they keep this special for very long."

After the meal, however, Meg complained, "I think this pizzeria is making more money than ever!"

Why was Meg so upset?

Hint 1

Draw a diagram comparing the sizes of the pizzas.

Hint 2

Solve an analogous problem. Suppose the pizzas were square shaped; what would be the effect of halving the length of one side?

Hint 3

Do a bit of research. Find a formula for calculating the area of a circle.

Answer: Meg was upset because she had to pay the same amount for the special but received only three-fourths as much pizza as she would ordinarily get.

23. Caveat Emptor

Consider the drawing of the regular and special cases below.

One mushroom and sausage pizza with a diameter of 50 cm

Three mushroom and sausage pizzas with diameters of 25 cm

It appears that even though the diameter has only been halved, three small pizzas will fit inside the one large pizza. What has upset Meg is that area, not diameter, is the better measure of the amount of pizza she is getting.

If the large pizza were a 50 cm square and the small pizzas were 25 cm squares, the area of the large one would be 2500 sq cm and the area of one small pizza would be 625 sq cm, only one-fourth the area of the large pizza. Thus, in this case, three small pizzas would only be three-fourths as much as one large pizza.

The area of a circle is πr^2 (the number π, which is approximately 3.14, times the square of the radius of the circle). In turn, the radius is $\frac{d}{2}$ (half the diameter of the circle).

Large Pizza	**One Small Pizza**
diameter = 50 cm radius = 25 cm	$d = 25$ cm $r = \frac{25}{2}$ cm
Area = π(25 cm) (25 cm)	$A = \pi(\frac{25}{2}\text{ cm})(\frac{25}{2}\text{ cm})$
Area = 625 π sq cm	$A = \frac{625}{4}\pi$ sq cm

The area of one small pizza is one-fourth the area of a large pizza. So the area of three small pizzas is three-fourths the area of one large pizza. Meg and Charlie paid the same amount ($5.00) but got only three-fourths the usual amount of pizza.