Problemoids

Grade 6
Math Mentor

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Implementation

The *Problemoids* program does not demand significant teacher involvement, although teachers may choose to participate quite actively. In either case the following recommendations will suffice for management of the programs.

1. ASSIGN THE PROBLEMS IN SEQUENCE.

Until the teacher becomes familiar with the problems, hints, answers, and solutions in *Problemoids,* we recommend assigning problems according to the order in which they appear in the student problem book. Generally, the position of the problem in sequence reflects its level of difficulty; that is, less difficult problems appear earlier and more difficult problems appear later in the sequence. Also, students' understanding of a problem appearing later in the sequence may be contingent on their familiarity with an earlier problem.

2. DIRECT STUDENTS TO MAINTAIN A RECORD OF THEIR WORK.

Students should maintain a record of their work toward solutions of problems. Space for work on each problem has been provided in the student book to assist students in organizing and keeping up with their solutions. The solution of a problem may be attempted over several sittings. For this reason, students need one fixed workspace to help them organize the solution and recall what they attempted in previous work.

If students do not work in their problem books, they should maintain their work in individual notebooks. A record that shows how individual problems were solved and how various strategies were employed may be beneficial for solving future problems.

3. DIRECT STUDENTS TO THE HINTS SECTION OF THE BOOK AND ENCOURAGE MODERATE USE OF THE HINTS.

If students feel blocked in their solution of a problem, they should be encouraged to use one or more hints. Hints to a given problem are not grouped together, but appear separately to encourage using hints one at a time. For your convenience, the teacher's manual has the hints grouped together. Students probably will obtain the most benefit from a hint by allowing themselves time for reflection instead of reading through all of the hints to a problem quickly. Since this factor is difficult to control, the hints have been designed to provide instructional value in either case.

4. ALLOW STUDENTS AMPLE TIME TO THINK ABOUT AND SOLVE EACH PROBLEM.

The authors suggest assigning one or two problems at the beginning of a week. Since many of the problems are quite challenging, students usually will not be able to solve an individual problem during one class period.

Students will need to spend time understanding a problem, investigating possible approaches to its solution, reflecting upon how to utilize hints, debugging unsuccessful solutions, and satisfying themselves that a proposed solution is correct. To accomplish this

students should be allowed to work on an assigned problem over several days and take their problem books home. At the end of the week students' answers can be corrected using the answers in *Problemoids: Math Mentor Grade 5*.

By design the problems in *Problemoids* present challenges; consequently, they have the potential to be a source of frustration to students whose major goal is producing correct answers. Although some students are well aware when they are making progress and are perfectly satisfied that their work will eventually result in an answer, other students need reassurance that the problem can be solved and that the hints may provide assistance. Reassurance may be enough to motivate these students to continue working.

5. ENCOURAGE STUDENTS TO READ AND EVALUATE SOLUTIONS PRESENTED ON SOLUTION SHEETS.

A solution sheet should be posted immediately after students' answers have been checked. It is important that students have an opportunity to read the solution sheet and compare the suggested solution(s) with their own solution. This procedure provides for self-checking of the most complex part of students' work and enables students to learn problem-solving strategies from the solutions whether *or not* they solved the problems correctly. Students should be encouraged to evaluate their solutions as correct, better than the authors' solution(s), or incorrect.

Students who believe they are close to a solution or at least on the right track may resist looking at a solution until they complete their own work. Such students might be allowed, within reason, more time to work on the problem before considering the authors' solution. A great deal of satisfaction and self-confidence can be gained from the knowledge that "I did it myself!" Seeing a solution prematurely might cause resentment and frustration. However, since the solutions have instructional value, it would be inappropriate to allow a student to build up a backlog of unsolved problems.

In the event that any students abandon a problem in frustration, it is doubly important that they compare what they have done with the solution(s) provided by the authors. They may find that they actually were on the right track *and that* they had made real progress toward obtaining an answer, and, most importantly, they may understand what they could have done to solve the problem.

NOTE: It is important to distinguish the difference between an answer to a problem and a solution to a problem. Answers are just that—answers; They are often very short and usually provide no information about the strategies used to solve the problem. A solution is the process by which an answer to a problem is obtained. Solutions may be lengthy, and they provide information about the strategies used.

Appendix III: Classification of Problems According to Mathematical Topic

● = primary classification ★= secondary classification	Sets	Number Numeration	Operations	Algebra	Geometry	Measurement	Probability & Statistics
1. Number Factory		●	★				
2. Germ Generation			●				
3. What-a-Question						●	
4. Emit on Time		★	●				
5. Great Rings						★	●
6. One to Ten		●	★				
7. Space Flash						●	
8. Dicing		●			★		
9. I-Deal Melons				★			
10 Double Vision					●		
11. Old Dog, New Trick							●
12. Equal Juice Amendment			★			●	
13. Saving Choice	●			★			
14. Space Chase		●	★	★			
15. Doubly Deep				●		★	
16. All for One			●	★			
17. Tucumcari		★					●
18. Anti-climatic		★					●
19. Identity Crisis	●						
20. Lost in Time			●	★			
21. Cutting Corners					●	★	
22. Sum Power		●	★				
23. Hurry Up and Wait			●	★			
24. Missing Link	●						
25. Dollars for Denim		●	★	★			

	Sets	Number Numeration	Operations	Algebra	Geometry	Measurement	Probability & Statistics
26. Strawberry Delirium			●	★			
27. Luck or License?							●
28. Foot Loose on the Track						●	
29. Rendezvous				★		●	
30. Quarter Moon							
31. Weigh to Win	★			●		★	
32. Downtown or Downhill?			★	●			★
33. Exponential Expertise		★	●				
34. The Whole Hole					●	★	
35. Annular Checkup				★	★	●	
36. Tipping the Scales	★						●
37. Divide and Conquer		★	●				
38. Short Way to Prey			★		★	●	
39. Dinosaur Derby							●
40. Fuel for Thought				★		●	
41. Quick Thinking, Good Job	●						
42. Clickety-Clack				★		●	
43. For Every Winner, There's a Loser							●
44. Prime Celebration		●					
45. Dollars and Sense				●			
46. Rug Ripper					●		
47. Leftovers		★	●				
48. Pick a Pair							●
49. Time Slips Away				★		●	
50. Germicide Squad			●	★			

26. Strawberry Delirium

Betty, Al, Jose, and Linda drove to the strawberry patch and picked 154 quarts of strawberries. Betty picked the most strawberries. She picked 4 more quarts than Al, 7 more than Jose, and 11 more than Linda. The charge for all the strawberries was $192.50. Betty knew that her share would be the most, but nobody knew exactly how much to pay. Calculate the fair share for each person.

Hint 1

Change the question to "How many quarts did each person pick?"

Hint 2

Change the information given to make each person pick as many quarts of strawberries as the person who picked the most.

Hint 3

Solve a simplified related problem. For example, if 15 quarts of strawberries were picked altogether, and Greg picked 1 quart more than Ann and 2 quarts more than Sam, how many quarts did each person pick? What is a simple rule for obtaining your solution?

Answer: Betty—$55.00, Al—$50.00, Jose—$46.25, Linda—$41.25.

26. Strawberry Delirium

You might have solved the problem in Hint 3 by the trial and error method, listing some of the possibilities. For example, we know that Greg, Ann, and Sam each picked a different number of quarts of strawberries and that those numbers are consecutive. Perhaps they were among the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9. Since we know that the numbers must sum to 15, inspection shows that 4, 5, and 6 are our numbers. This approach does not lend itself readily to more complex problems, but you could re-examine these results in an attempt to develop a more generally applicable approach to problems of this nature.

Suppose we kept the same number of people, three, and temporarily changed the information given so that each person picked as many quarts of strawberries as the person who picked the most, Greg. Now, the total number of quarts of strawberries picked would have been 15 quarts plus 1 quart (to make Ann's number equal Greg's) plus 2 quarts (to make Sam's number equal Greg's), or 18 quarts. Greg would have picked 18 quarts divided by 3, or 6 quarts. Then Ann would have picked 1 quart less, or 5 quarts, and Sam would have picked 2 quarts less, or 4 quarts.

Thus, a simple rule or procedure for solving such problems might be: Add each of the differences from the highest number to the total number of items and divide by the number of participants to find the number of items generated by the highest contributor. Then subtract the differences one at a time from the number of items generated by the highest contributor to calculate the number of items generated by each participant.

In our problem Betty was the highest contributor; consequently, we add to the total of 154 quarts of strawberries 4 quarts difference for Al, 7 quarts difference for Jose, and 11 quarts difference for Linda, for a total, of 176 quarts. Betty picked 176 quarts divided by 4, or 44 quarts. Al picked 4 quarts less, or 40 quarts; Jose picked 7 quarts less, or 37 quarts; and Linda picked 11 quarts less, or 33 quarts. The price of strawberries per quart is \$192.50 divided by 154 quarts, or \$1.25 per quart. Hence:

Betty's share is \$1.25/qt. $\times$ 44 qts. = \$55.00

Al's share is \$1.25/qt. $\times$ 40 qts. = \$50.00

Jose's share is \$1.25/qt. $\times$ 37 qts. = \$46.25

Linda's share is \$1.25/qt. $\times$ 33 qts. = \$41.25

Looking Back

You usually should check to make certain that the answer you produce fits all the conditions of the problem. If you produced an answer to this problem and it was not correct, check to see if your answer fits all the conditions of the problem. Sometimes you may produce an incorrect answer because you ignore or misinterpret some condition or because you do not check whether your answer satisfies all the conditions stated in the problem.