

Problemoids

Grade 4
Math Mentor

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Introduction

Problemoids is a problem-solving program designed to meet the needs of intellectually gifted students and other students who may benefit from an enriched mathematics program more advanced than that provided in the standard curriculum. *Problemoids* engages students in high-level thinking about challenging problems and provides a stimulating opportunity for students to develop and increase their repertoires of problem-solving strategies.

Problemoids Level 4 contains 36 problem sets with five problems in each set. Altogether, there are 180 problems with solutions. The problems differ from those in the typical curriculum in that solving them requires one or more mental abstractions. A separate book is available for students.

The problems in *Problemoids* require students to use the full spectrum of thinking skills in Bloom's Taxonomy with special emphasis on high-level skills such as analysis, synthesis, and evaluation. The mathematical skills required of students are appropriate for grade 4. Topics covered in the program extend, enrich, and reinforce topics typically covered in the mathematics curriculum: sets, number and numeration, operations, geometry and measurement, algebra, and probability and statistics.

Strategy-directed hints accompany the main problem in each set to assist students in solving the problem and in learning problem-solving strategies. Solutions to main problems emphasize problem-solving strategies suggested in the hints by showing how to solve the problems using those strategies.

Students obtain maximum benefit by participating at all available levels of the *Problemoids* program; yet students can participate at any level without prior exposure to the program.

GOALS

The main goals of the program include:

- (1) Improving students' problem-solving performance
- (2) Enriching students' mathematics program by teaching problem-solving strategies
- (3) Assisting students in creating a "memory bank" of problems, solutions, and methods of solutions

OTHER OBJECTIVES INCLUDE:

- a. Exercising students' high-level thinking skills by challenging them with problems that require use of analysis, synthesis, and evaluation on a regular basis
- b. Fostering a sense of self-confidence in solving problems
- c. Extending students' perseverance in attempting solutions to challenging problems
- d. Providing experiences that may serve to increase students' interest in mathematics

PROGRAM CONTENT

The program is organized into 36 sets of problems. Altogether there are 180 individual problems. Less-difficult problems appear earlier in the book; more difficult problems appear later. Each problem set includes a main problem and four related problems called “looking back questions.” The solutions to looking back questions are related to the solution methods of the main problem in the set.

Problems are designed to provide experience in choosing and using problem-solving strategies. The problems cannot be solved by simply choosing an operation such as addition, subtraction, multiplication, or division. The following list indicates the strategies that are suggested for solving problems in this book. Students may develop different terminology for various strategies—that’s fine.

Restate the Problem in Your Own Words

What Information Is Given in the Problem?

What Are the Conditions?

What Are You Asked to Find?

Trial and Error

Use All Given and Implied Information

Solve Part of the Problem

Draw a Diagram

Make a List or Make a Chart

Search for a Pattern Solve a Simpler Problem

Work Backward

As many as four strategy-directed hints accompany the main problem in each set. The first looking back question in each set includes a strategy-directed hint. Most other looking back questions do not; instead, they provide opportunities for students to think of strategies themselves.

SET THE TONE

Many factors affect the success of a problem-solving program. Some can be controlled more than others. The classroom atmosphere is one of the most important aspects affecting success, and it depends largely on the actions and the attitude of the teacher.

During the first eight to ten weeks of the program, the teacher’s emphasis should be on classroom atmosphere. Paying careful attention to the development of a supportive atmosphere early can accelerate progress for the remainder of the year. Give special emphasis to the value of taking risks. Compliment children for playing hunches, and encourage others to be supportive. Your consistent enthusiasm and praise can make an enormous difference.

Classification of Problems According to Mathematical Type

● = primary classification
 ★ = secondary classification

	Sets	Number Numeration	Operations	Algebra	Geometry & Measurement	Probability & Statistics
1. Lucky Combination		●	★			
2. Century Toss	★		●			
3. Have It Your Way						●
4. A Fishy Story			★	●		
5. Pacific Puzzler		●	★			
6. Sticky Addition		●	★			
7. Stairway to the Top		●				
8. Pennant Fever		★			●	
9. Seven for Sisters	●			★		
10. A Few Are Due			●	★		
11. Crazy Cube		●			★	
12. Computer Survey			●	★		
13. Mean Scream						●
14. An Odd Tee Shirt		★		●		
15. Box Lunch			★	●		
16. Pizza Pieces		★			●	
17. Penny Pincher				●		
18. Ups and Downs			●	★		
19. Check the Corners	★				●	
20. Shelling Out Seashells	★	●				
21. Taste Test			●	★		
22. Raising Questions			●			
23. SOS			●	★		
24. Tower of Triangles		★			●	
25. Engineering a Fraction		★	●			

	Sets	Number Numeration	Operations	Algebra	Geometry & Measurement	Probability & Statistics
26. Join Up		●		★		
27. Serving the Class			●	★		
28. Can Clean			●			
29. Comic Question			●	★		
30. Sporty Product		★	●			
31. Dottygons		●			★	
32. Raking It In			●	★		
33. Dirty Fractions			●		★	
34. Small Profits			●			
35. Shangri-La		●	★			
36. Running for the Gold			●			

15. Box Lunch - Solution

Use All Given And Implied Information. Since the lunch box with the monster burger costs \$7.00 and the lunch box with the plain burger costs \$5.00, how much more does the monster burger cost than the plain burger? Then how much must the plain burger cost? Then how much does the lunch box cost?

The monster burger must cost $\$7.00 - \$5.00 = \$2.00$ more than the plain burger. Since the monster burger costs three times as much as the plain burger, the plain burger must cost \$1.00. Then the lunch box by itself must cost $\$5.00 - \$1.00 = \$4.00$.

Use Trial and Error and Work Backward. Guess how much the lunch box costs. Then figure how much the plain burger must cost and how much the monster burger must cost according to your guess. Check whether your guess is correct using the conditions of the problem. *Make a Chart* to keep track of your guesses. For example:

Guess cost of lunch box	Then find cost of plain burger	Then find cost of monster burger	To check, ask yourself, "Is three times the cost of the plain burger equal to the monster burger?"
\$1.00	$\begin{array}{r} \$5.00 \\ -1.00 \\ \hline \$4.00 \end{array}$	$\begin{array}{r} \$7.00 \\ -1.00 \\ \hline \$6.00 \end{array}$	$\begin{array}{r} \$4.00 \\ \times 3 \\ \hline \$12.00 \end{array}$ No, \$12.00 does not equal the cost of monster burger.
Guess again \$4.00	$\begin{array}{r} \$5.00 \\ -4.00 \\ \hline \$1.00 \end{array}$	$\begin{array}{r} \$7.00 \\ -4.00 \\ \hline \$3.00 \end{array}$	$\begin{array}{r} \$4.00 \\ \times 3 \\ \hline \$12.00 \end{array}$ It checks!

We could guess that the lunch box costs \$1.00. Then the plain burger would cost $\$5.00 - \$1.00 = \$4.00$. The monster burger would cost $\$7.00 - \$1.00 = \$6.00$.

To check our guess we see if the monster burger costs three times as much as the plain burger: $3 \times \$4.00 = \12.00 not \$6.00, so that guess is incorrect.

Guessing again we try \$4.00 for the cost of the lunch box. Then we find the cost of a plain burger must be \$1.00, the cost of the monster burger must be \$3.00, and three times \$1.00 does equal \$3.00. It checks, and the cost of the lunch box is \$4.00.

15. Box Lunch - Solutions to Looking Back Questions

- (A) The monster burger costs $\$7.00 - \$4.50 = \$2.50$ more than the plain burger. Since the monster burger costs three times as much as the plain burger, the plain burger must cost $\$2.50 \div 2 = \1.25 and the monster burger must cost $3 \times \$1.25 = \3.75 . Therefore, the lunch box must cost $\$4.50 - \$1.25 = \$3.25$. A check of our calculation using the monster burger prices also shows that the lunch box costs $\$7.00 - \$3.75 = \$3.25$.
- (B) The chocolate icing costs $\$8.30 - \$6.50 = \$1.80$ more than the sprinkles. So the sprinkles must cost $\$1.80 \div 2 = \0.90 and the chocolate icing must cost $3 \times \$0.90 = \2.70 . Therefore, the yellow cake with no icing must cost $\$8.30 - \$2.70 = \$5.60$. A check of our calculation using the yellow cake and sprinkles prices also shows that the yellow cake costs $\$6.50 - \$0.90 = \$5.60$.
- (C) The chocolate icing costs $\$8.30 - \$6.50 = \$1.80$ more than the sprinkles. So the sprinkles must cost $\$1.80 \div 3 = \0.60 and the chocolate icing must cost $4 \times \$0.60 = \2.40 . Therefore, the yellow cake with no icing must cost $\$8.30 - \$2.40 = \$5.90$. A check of our calculation using the yellow cake and sprinkles prices also shows that the yellow cake costs $\$6.50 - \$0.60 = \$5.90$.
- (D) If the Scottie dog savings bank costs the same amount as the music box, the bank and the music box would have cost $\$8.30 - \$1.00 = \$7.30$ instead of $\$8.30$. This way the bank and the music box would have cost $\$7.30 \div 2 = \3.65 each. Since the Scottie dog bank actually costs $\$1.00$ more than the music box, the music box costs $\$3.65$ and the Scottie dog savings bank must cost $\$3.65 + \$1.00 = \$4.65$.