Awesum Alex
Math Detective

The Addition Problem

Implementation Manual

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Arithmetic is answering the question.
Mathematics is questioning the answer.

"That isn't how we did it when I went to school." That statement is one made by many adults when they attempt to help a child with his or her math homework. And although it's a true statement, one could also say, "This isn't the same phone I had when I went to school," or "This isn't the same TV we had when I was in fifth grade."

Here's another one: "It was good enough for me so, it's good enough for my child/my students." Well, not necessarily. In today's world, we are educating our children for a job market that likely will be radically different fifteen or more years from now when they finally get there. If we are interested in the next generation being mathematically astute, then the teaching of mathematics has to change with the times.

Most of us learned math using the "know how" method, which is known as *procedural knowledge*. Procedural knowledge is using well-defined rules or steps to obtain a result or a quick correct answer. This method involves rote memorization but not necessarily understanding, and therefore it leads to underdeveloped pieces of knowledge. When a learner tries to memorize procedures, rules, or routines without understanding them, the information is quickly forgotten or even feared.

Conceptual knowledge is the "know why," and in mathematics, it can be thought of as a deep understanding of the mathematical ideas—the big ideas that cannot be learned by rote memorization.*

When children learn to add through a conceptual understanding of place value and algebraic reasoning, they begin to understand the power of mathematics. For elementary learners to have a conceptual knowledge of the math, they must be taught the breadth, depth, and thoroughness of the subject. If the instructor does not have that understanding, then he or she can't teach it. Adults cannot empower students with mathematical competence if they do not have it themselves.

* The concept of knowing *how* versus knowing *why* comes from Chinese teacher, principal, and author Liping Ma in her 1999 book *Knowing and Teaching Elementary Mathematics*.

The cyclical problem in educating children is instructors who cannot teach what they do not know. Consequently, instructors revert back and teach how they have been taught. If they were taught only mathematical procedural knowledge, then that's what they will teach, and conceptual knowledge will continue to elude their students. Somewhere along the lines of mathematical education, a paradigm shift must occur. Instructors must understand and be able to teach the essential standards of mathematical proficiency, including the following:

1. The multiple ways of representing numbers
2. The fluency and flexibility of numbers
3. The place value base ten number system
4. The connection of basic arithmetic to algebra

For early learners to have conceptual knowledge about mathematics, it must be experiential—that is, the numbers must be put in context. One cannot have four; it must be four of something, such as four fingers, four cats, or four markers. Numbers, when used mathematically, are adjectives, not nouns. This is important when teaching children the basic addition operation. Three and four cannot be added, since the common term is not known. Nor can unlike or dissimilar terms be added. Three dogs cannot be added to four houses to get seven doghouses. Only like or similar terms can be added or subtracted, and this has implications later on in algebra. Three chairs can be added to four chairs, and the result is seven chairs. This is why a common denominator is used for adding or subtracting fractions—it is adding or subtracting like or similar terms.

For children to think conceptually about the basic arithmetic operations, they have to be taught to think creatively about numbers, which means they have to have number fluency and number flexibility. They must learn the difference between *expressing*, *decomposing*, and *partitioning* a number, along with learning how to write a number in many forms.

Thinking Creatively about Numbers

Expressing a number is a way of saying the number mathematically through any math operation without using the number itself. There are infinite ways to express a positive or a negative number using all of the mathematical operations available.

Examples of ways to express a positive 12 (+12):

$+12 = +14 - 2$

$+12 = +6 + 2 + 2 + 2$

$+12 = +2^2 (+3)$

$+12 = +\sqrt{144}$

$+12 = +13 - \frac{3}{4} - \frac{1}{4}$

$+12 = +\sqrt{25} + 3^2 - 2$

Examples of ways to express a negative 12 (-12):

$-12 = -11 - 1$

$-12 = -13 + 1$

$-12 = -3 (+4)$

$-12 = -10 - 2$

$-12 = -3 - 3 - 3 - 3$

$-12 = -14 + 1 + \frac{1}{2} + \frac{1}{2}$

$-12 = -\sqrt{25} - \sqrt{81} + \sqrt{4}$

Decomposing a number is the process of breaking the number down into other parts through addition or subtraction. You can use negative numbers and fractions in this process. There are infinite ways to decompose numbers.

Examples of decomposing a positive 12 (+12):

$+12 = +4 + 4 + 4$

$+12 = +8 + 2 + 2$

$+12 = +15 - 3$

$+12 = +7 + 4 + \frac{1}{2} + \frac{1}{2}$

$+12 = +2^3 + 2^2$

$+12 = +3! + 3!$ (factorials)

$+12 = +12 + 1 - 1$

$+12 = +10 + 2.7 + 0.3$

$+12 = +9 + 6 - 2 - \frac{1}{2} - \frac{1}{2}$

Examples of decomposing a negative 12 (-12):

$-12 = -10 - 2$

$-12 = -14 + 1 + 1$

$-12 = +13 - 20 - 5$

$-12 = -5 - 5 - 2$

$-12 = -20 + 10 - 2$

$-12 = -15 + 1.5 + 1.5$

Partitioning a number is the process of decomposing a positive whole number using only whole numbers and addition. Partitioning numbers is a subset of decomposing numbers.

When partitioning a number, it is best to write the addends in descending order. *Descending order* is when numbers are arranged from the largest to the smallest, left to right.

Examples of partitioning a positive 12 (+12):

$+12 = +12 + 0$

$+12 = +11 + 1$

$+12 = +10 + 2$

$+12 = +10 + 1 + 1$

$+12 = +9 + 3$

$+12 = +9 + 2 + 1$

$+12 = +9 + 1 + 1 + 1$

$+12 = +8 + 4$

$+12 = +8 + 3 + 1$

$+12 = +8 + 2 + 2$

$+12 = +8 + 2 + 1 + 1$

$+12 = +8 + 1 + 1 + 1 + 1$, and so on

As you can see, there is only a finite number of solutions for partitioning numbers.