Awesum Alex
Math Detective

The Subtraction Problem

Implementation Manual

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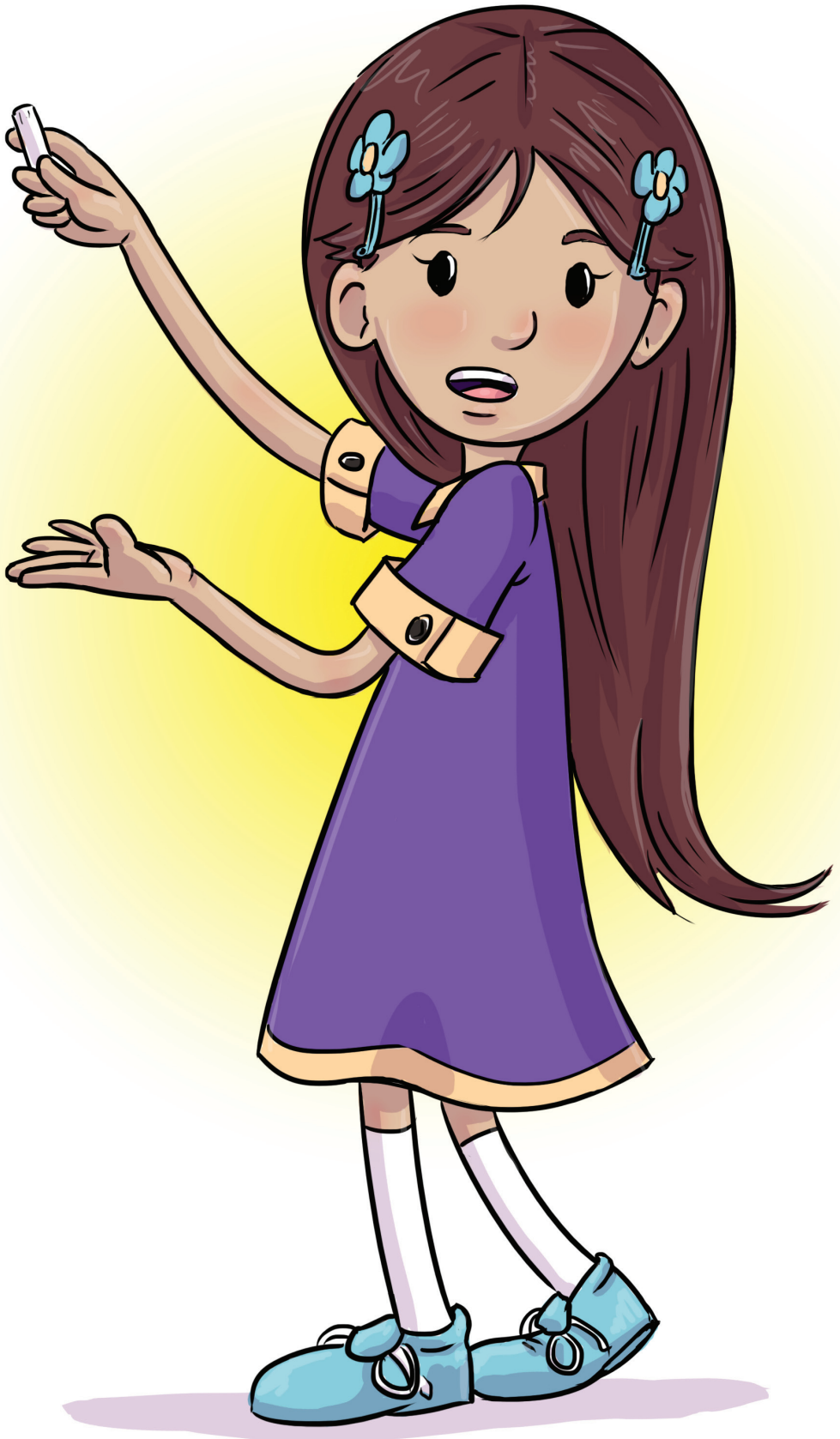

Changing How We Make Change

Carol and Rachel, the authors of this manual, were discussing their earliest memories of how to make money change. Making change used to be done manually before the age of computers and credit cards. Carol remembers going to the store and watching as her mother handed a bill or several bills and change to the store clerk and received bills and/or change back. "I was always baffled at how the clerk knew the exact change to give my mom," Carol recalls. "I wondered how she subtracted so quickly in her head and how my mom always knew what to expect back."

Rachel, who went to a one-room schoolhouse, remembers her third-grade teacher, Mr. Stamm, who was always jingling coins in his pocket. Rachel explains, "Every day, Mr. Stamm would place a bunch of change on my desk and give me a change-making problem. He would tell me, for example, that he had bought something for sixty-three cents and had given the clerk a dollar bill, and he would ask me to make the change for the dollar bill. I would put two pennies in his hand, counting, 'sixty-four cents, sixty-five cents,' then place a dime in his hand and say, 'seventy-five cents,' and finally put a quarter in his hand and say, 'one dollar.' He would then ask me to count the total amount of change that he would receive. I would count the change starting with the quarter, saying, 'Twenty-five cents plus a dime equals thirty-five cents plus two cents is a total of thirty-seven cents.' He would do this several times a day, probably to keep me quiet and out of trouble."

Making change manually is different from counting change. When counting change, one starts with the larger coins first, which is the opposite of making change, when one starts with the smaller coins first.

Subtraction can be defined as the operation of removing objects from a given set. It can also be defined as removing a part from a whole. There are several mathematical skills that children must have before they can subtract. The first skill is understanding *parts of a whole*. The second skill is *counting on*, starting at any number. The third skill is *counting by tens*, starting at any number.

When children are taught to count, the emphasis is always placed on the total amount of what they are counting. Let's suppose that a child is counting crayons, and there are a total of six of them on her desk. The child will count, "One, two, three, four, five, six." The adult may ask, "How many crayons are there?" If the child answers, "four," she will be told that she is incorrect. But the child's answer of four *is* correct, since four is contained within six. The more precise question that should have been asked is: "What is the total number of crayons on the desk?"

As adults, we assume that when a student counts six of something, she also understands that within that six, she has one and two and three and four and five of that item. As parents and teachers, we tend to emphasize the total and forget to work on the parts. If the child does not understand that three crayons are contained in six crayons, then how is she to understand that you can subtract three crayons from six crayons? She must know that if she counts six crayons, then she still has three crayons. This concept is called *parts of a whole*, and it is fundamental to our understanding of subtraction. What follows are some basic activities to help foster this understanding.

Suggested activities:

1. Lay out four spoons, and ask:

    a. How many spoons are there in this entire group?

    b. Is there one spoon in the group?

    c. Are there two spoons in the group?

    d. Are there three spoons in the group?

    e. Are there four spoons in the group?

    Questioning this way should lead to an understanding that the spoons are all parts of the whole group.

2. Now let's proceed to money. Take out ten one-dollar bills, and ask:

    a. Is there a one dollar in this group?

    b. Are there two dollars in this group?

    c. Are there three dollars in this group?

    d. Are there four dollars in this group?

    ...and so on. An understanding of parts of a whole should follow.

3. Exchange the ten one-dollar bills for one ten-dollar bill, and have the child look for what is hidden there. Follow with the same line of questioning.

4. Next, introduce fractions.

    a. Take one cookie (or something similar), and ask: "Is there a half of a cookie here?" (Be prepared for a "no.")

    b. Take a knife, and cut the cookie into two equal pieces. Ask, "Is there half here?" Put the two pieces together, and ask again.

    c. Take out another whole cookie, and ask the question again. Explain that one does not have to cut the cookie in half for the half to be there.

    d. Take out three cookies, and ask, "How many whole cookies are here?"

    e. Then ask, "If I were to cut the cookies in half, how many half cookies would there be?" Have the child count the cookies by halves: "one half, two halves, three halves," etc.

5. Encourage the child to choose several items to practice with. These may include blocks, pieces of paper, spoons, or other objects. Practice the "parts to whole" concept with halves, thirds, fourths, etc. Reinforce this while eating breakfast (using pancakes, for example, or toast) and lunch (using sandwiches, perhaps).

Counting on is the second skill that children must learn to become proficient at simple mental subtraction. Many times children know how to count to

one hundred, but when they are asked what number comes after twenty-two, they have to start at one and count up to twenty-two to get the answer. It is more important, however, that children know what number comes before and after a particular number up to ten than it is that they memorize the numbers up to one hundred. Once they can count to ten, then they can apply that knowledge to the numbers up to twenty, then thirty, and so on.

Suggested activities:

1. Begin by giving the child a single-digit number, such as five, and asking him to say the number before and after it (the child will say, "Four, five, six."). Take turns doing this with the child, which will allow him both to give a correct answer and to hear a correct answer. Once he is proficient with single-digit numbers, he can move on to double-digit and then triple-digit numbers.

2. Say a number, and have the child give the three consecutive numbers after it. For example, if you say, "Twenty-four," the child should respond with, "Twenty-five, twenty-six, twenty-seven." Take turns doing this with the child. Start with a single-digit number, and progress to double-digit numbers and then triple-digit numbers.

It is important for children to understand that when they are counting forward, they are adding one to the number before; basically, they are counting by ones. When they are counting backwards, they are subtracting by one. Whenever children are counting by twos, they are actually adding twos. When they are counting by threes, they are adding threes, and so on.

Suggested activities:

1. Have the child practice counting by ones starting at 1 (make sure she knows that she is adding one each time):

   1 2 3 4 5 6 7 8 9 10...

   +1 +1 +1 +1 +1 +1 +1 +1 +1...

   Count by ones starting at 8:

   8 9 10 11 12...

   +1 +1 +1 +1...

Count by ones starting at 47:

47 48 49 50 51...

+1 +1 +1 +1...

Count by ones starting at 103:

103 104 105 106 107...

+1 +1 +1 +1...

2. Have the child practice counting by twos starting at 2 (make sure she knows that she is adding two each time):

   2 4 6 8 10 12...

   +2 +2 +2 +2 +2...

   Count by twos starting at any multiple of two:

   8 10 12 14 16 18...

   +2 +2 +2 +2 +2...

   Count by twos starting at an odd number:

   3 5 7 9 11 13 15 17 19...

   +2 +2 +2 +2 +2 +2 +2 +2...

3. Have the child practice counting by threes starting at 3 (make sure she knows that she is adding three each time):

   3 6 9 12 15 18...

   +3 +3 +3 +3 +3...

   Count by threes starting at any multiple of three.

4. Then by fours:

   4 8 12 16 20 24...

   +4 +4 +4 +4 +4...

   Count by fours starting at any multiple of four.

5. Then by fives:

   5 10 15 20 25 30...

   +5 +5 +5 +5 +5...

   Count by fives starting at any multiple of five.

Challenge activity:

Have the child count by threes, fours, and fives starting at numbers other than a multiple of that number. For example:

1. Count by threes starting at 7: 7, 10, 13, 15....
2. Count by fours starting at 15.
3. Count by fives starting at 8.

Counting by tens is the third skill necessary for mental subtraction. Most parents and teachers teach children to count by ten starting at ten. However, children should learn how to count by tens starting at any number. They should learn to look at the patterns when counting by tens—they need to understand that when they are counting by tens, they are adding ten to the previous number.

Suggested activities:

1. Explain to the child: "Every time I ask you to count by ten, I want you to ask me, 'Starting where?'"
2. Then say, "Right now, let's start with the number three."
3. Lay out three dollars, and then lay out ten more dollars. Ask, "How much do you have?"
4. Show with pictures and numbers: Three add ten equals thirteen; add another ten equals twenty-three; add another ten equals thirty-three; add another ten equals forty-three, and so on.
5. Start with seven, and have the child go up to forty-seven counting by tens. Switch roles with the child; let the child become the instructor, and you become the learner.