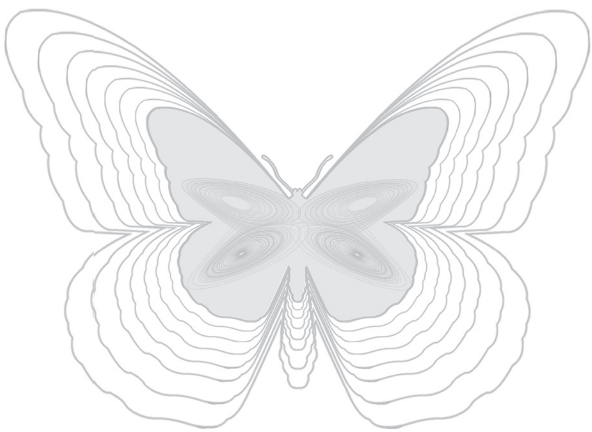
Mathematical Lives

EDWARD LORENZ

AND THE CHAOTIC BUTTERFLIES



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Royal Fireworks Press
Unionville, New York

Also by Robert Black,
published by Royal Fireworks Press:
The Mathematical Lives series
The Mathematical Nights series
Liberty Girl
Unswept Graves
The Eyes of the Enemy

Royal Fireworks Press
P.O. Box 399
41 First Avenue
Unionville, NY 10988-0399
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email: mail@rfwp.com
website: rfwp.com

ISBN: 978-0-88092-785-7

Publisher: Dr. T.M. Kemnitz
Editor: Jennifer Ault
Book Designer: Kerri Ann Ruhl
Cover Illustrator: Christopher Tice

Printed and bound in Unionville, New York, on acid-free paper using vegetable-based inks at the Royal Fireworks facility. 17ja22

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Prologue

The Butterfly Effect

No matter how advanced our science and technology become, there are forces of nature that we can neither escape nor control. And among those forces of nature, weather is probably the one we think about most. After all, weather can affect so many parts of our lives—matters as small as whether we can play outside this afternoon or as large as whether a country will have enough food to eat.

That's not to say that people haven't *tried* to control the weather, or at least to predict what was coming their way. In ancient times, civilizations held ceremonies to appease the gods and goddesses they believed were in the sky, controlling their fates. Priests, shamans, and other mystics practiced elaborate rituals to reveal what those gods and goddesses had in store for them. In more recent times, mysticism gave way to science, from the inventions of the thermometer and the barometer to the construction of weather observation stations connected first by telegraph and then by telephone and radio. People were no closer to controlling the weather, but at least they could warn one another about what was coming.

In the 1950s, though, it began to look like the weather might finally be conquered. The launching of the first artificial satellites showed that it was possible to put "eyes in the sky" that were capable of observing weather patterns on a global scale. At the same time, the invention of electronic computers made it possible to perform the large number of calculations needed to model the atmosphere's behavior. In 1959, Walt Disney Studios produced a short film that predicted the creation of a global weather forecasting and control organization that used the latest technology to protect people from natural disasters.

But just one year later, one mathematician made a discovery that smashed all of those dreams forever. And he did it by accident.

Edward Norton Lorenz was a professor of meteorology at the Massachusetts Institute of Technology, commonly known as MIT. In his office sat one of the first "small" computers, a Royal-McBee LGP-30, which was the size of a large desk but which had a memory of only 4,096 32-bit words. He was using it to run a weather simulation program, calculating values for twelve different variables in increments of six hours to look for patterns over a long period of time. Each set of values was printed on a page of computer paper at a rate of one simulated day per minute.

One day, while preparing for an upcoming conference, Lorenz decided to rerun part of his simulation, starting over from the middle of a previous run. He went to his computer, typed in the values of each variable at the point he wanted, and then went to get a cup of coffee while he waited for the results. When he returned, he was surprised to find the computer printing a completely different set of numbers from the ones it had calculated before. The equations were the same, so the results should have been the same as well. Why weren't they?

At first Lorenz suspected a problem with the computer—something that occurred often in the early years of the Computer Age. But a closer examination revealed the answer. In order to fit all twelve variables onto a single line of text, Lorenz had programmed the computer to round each value off to only three decimal places. In the computer's memory, though, the variables were calculated and stored to six decimal places. So when Lorenz typed in the values he wanted to use in order to rerun the simulation, he hadn't entered the *exact* same numbers. Imagine doing that yourself—typing "0.123" when the real value is 0.123456. That tiny difference was enough to send the results in a completely different direction.

The investigation that Lorenz and others made of the mathematics behind what had happened produced a new field of science: *chaos theory*. They proved that tiny changes in the starting conditions of a *dynamical system* (a system of objects in motion) can produce large differences in the final results. That means that accurate long-term predictions of something like the weather are impossible; the system is too sensitive to small changes.

In 1972, Lorenz gave a talk at the American Association for the Advancement of Science's annual meeting in which he compared his discovery to a butterfly in Brazil flapping its wings, and the disturbance in the air growing until it produces a tornado in Texas. The idea caught the public's imagination, and in popular culture it has become known as "the Butterfly Effect." But while that expression is familiar to many, Lorenz himself is still relatively unknown.

The path that Lorenz took to reach his discovery had some small changes that led to big consequences, too. Meteorology wasn't even his original career field but instead was one that circumstances brought him to. His story begins in a small New England suburb at a time when "chaos" was a good way to describe much of the world.

Chapter One

Math Puzzles and Games

May of 1917 was anything but peaceful. The United States had decided to enter World War I just a month earlier, and across the country, the government was trying to equip and train the thousands of young men it planned on sending into battle. The war had already been raging in Europe for almost three years. On the Western Front, the two sides were locked in a horrific stalemate, while in the East, German victories had resulted in the overthrow of Russian Tsar Nicholas II. And within a year, an influenza pandemic (called “Spanish influenza” but more likely to have started in Kansas) would sweep around the globe, killing millions.

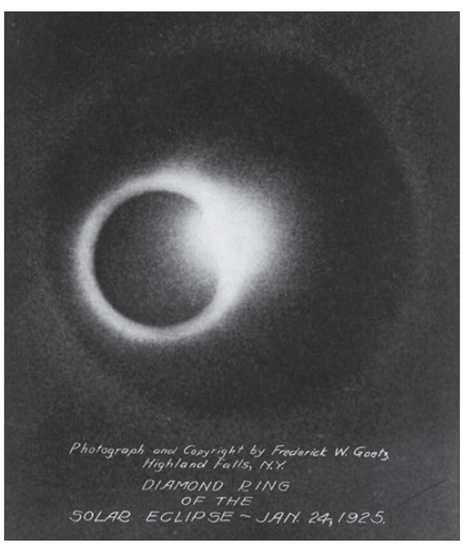
But in the small town of West Hartford, Connecticut, things were peaceful enough for Edward Henry Lorenz and his wife, Grace Norton Lorenz. They had married a year earlier and were expecting their first child. Edward Norton Lorenz was born in their home on May 23. A younger sister, Margaret, was born a little more than two years later, in July 1919.

The Lorenz household had deep mathematical and scientific roots. Edward’s father had earned a degree in mechanical engineering from MIT, where he also held the school track and field record for the two-mile run. By 1920, he was designing machinery that made bottles and other glass products at the Hartford-Fairmont Company, which still exists today as Bucher Emhart Glass. He remained at that job for his entire career, but in his free time, he liked to learn about other areas of science, and especially mathematics.

Edward's mother, Grace, had an MIT connection of her own. Her father, Lewis Mills Norton, had been a professor in MIT's chemistry department. In 1888, inspired by his knowledge of the German and British chemical industries, Norton introduced a new curriculum that he called "Course X," the first four-year chemical engineering program in the world. Grace's mother, Alice Peloubet Norton, was an academic as well. When Lewis died in 1893, she began teaching "Domestic Science" in various women's schools and studying "sanitary chemistry," earning her own MIT degree. Eventually, she moved her children to Chicago, where she taught at the University of Chicago School of Education and worked with the social services organization Hull House. Grace had been a teacher before her marriage, and she served on the West Hartford town council and school board once Edward and Margaret were born.

With that kind of background, it was no surprise that Edward took an interest in math from an early age. In fact, before he was two years old, he could read all of the numbers on the houses as his mother took him for walks in his stroller. After he learned the multiplication tables, he took an interest in perfect squares—numbers that were the product of a number multiplied it by itself. At one point he could recite the entire series of them from 1 to 10,000 (which is 100 times 100, in case you've forgotten).

By the time he was seven, Edward had added maps to his list of interests. He even drew his own maps of imaginary lands he'd invented. One day, while visiting friends of the family on their farm east of Hartford, he found an atlas among the books there and began to read it. After going through the maps of places around the world, he came to a page filled with different circular objects, including one that had a large ring around it, reminding him of a hat he'd seen in a cartoon. His father explained that the circles were the planets, including Saturn, which at the time was the only planet known to have rings. The discovery gave Edward a lifelong passion for astronomy—one that grew a few months later when he saw a total solar eclipse that was visible across the northeastern U.S.



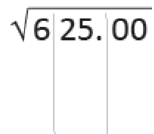
Throughout Edward's childhood, the Lorenz household was filled with all types of games. The family had a collection of jigsaw puzzles and would compete to see who assembled them the fastest, recording their times on the insides of the boxes. One set of those puzzles, about twenty with hand-cut wooden pieces, remained with Edward his entire life. He also enjoyed card games and board games, especially chess. By the time he reached high school and college, he was skilled enough to be captain of the school chess team. And of course, he loved mathematical puzzles, which he and his father often solved together.

One of those mathematical "puzzles" might seem strange to us today. Edward enjoyed taking square roots by hand—that is, finding the number that, when multiplied by itself, produces the number you started with. Computers took over that task decades ago, and today you can find a square root just by tapping the $\sqrt{\ }$ button on your calculator app, but there's a method for doing it with a pencil and paper.

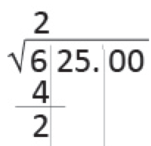
We won't get into the algebra of why this method works, but we can still try working a couple of examples. Let's start with an easy one and try a perfect square:

$$\sqrt{625.00}$$

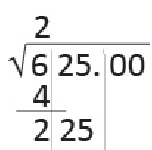
The first thing we need to do is group the digits by twos, starting at the decimal point and working to the left. For this problem, we group the 2 and the 5 together and leave the 6 by itself. Because 625 is a perfect square, we don't really need any digits to the right of the decimal point, but we'll leave the 00 there anyway. Some people draw faint lines down the page between the groupings to help them keep track of the number placements.



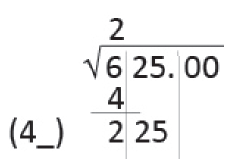
Let's look at the number 6. What's the largest perfect square less than it? It's 4, or 2^2. We therefore will write a 2 on the line above the 6 and a 4 below the 6. Then we'll subtract the 4 from the 6, leaving 2.



So far it looks just like a long division problem, but that's about to change. In long division, we would bring down the next digit, 2, next to the remainder, generating the number 22. But for a square root, we're going to bring down both digits in the next group, the 2 and the 5, making 225.



And now things are going to get even more different. Off to the left, we're going to write a note for ourselves: (4_).



What does that mean? The 4 is twice the value of the 2 in our solution, and the blank is the next value we need to find. The next digit in our solution will also be the digit we place in the blank so that forty-something times that something gives us the largest value less than 225. If we were expressing it algebraically, we would write $(40 + x)x \leq 225$, or $x^2 + 40x - 225 \leq 0$.

It's somewhat like a puzzle. Unless you want to do the algebra, you've got to try different digits until you find the one that fits. Perhaps that was what Edward found so appealing. In any event, we'll soon discover that 45 times 5 equals 225 exactly, leaving no remainder. Thus, the final value for the square root of 625 is 25.

```
         2  5
        √6|25.|00
         4
(45)     2|25
         2|25
            0
```

Most numbers, however, are not perfect squares. Finding their square roots is more difficult. Let's double our original number, 625, and find the square root of 1,250.

$$\sqrt{1250.00}$$

As before, we'll separate the digits into groups of two, and this time we'll add extra zeros to the end; we'll need them.

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√12|50.|00|00|00
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