Problemoids

Grade 5
Math Mentor

Revised Edition

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Introduction

Problemoids is a problem-solving program designed to meet the needs of intellectually gifted children, as well as those children who would benefit from an enriched mathematics program that is more advanced than that provided in the standard curriculum. It engages children in high-level thinking about challenging problems and provides a stimulating opportunity for them to develop and increase their repertoire of problem-solving strategies.

Problemoids Level 5 contains 50 problems, which generally increase in difficulty as children progress through the book. The problems differ from those in a typical curriculum in that solving them requires children to use the full spectrum of thinking skills in Bloom's Taxonomy, with special emphasis on high-level skills such as analysis, synthesis, and evaluation. The topics covered in the program extend, enrich, and reinforce topics typically covered in a fifth-grade mathematics curriculum: sets, numbers and numeration, operations, algebra, geometry, measurement, and probability and statistics.

As many as four strategy-directed hints accompany each problem to assist children in solving the problem and in learning problem-solving strategies. The solutions emphasize those strategies by illustrating how to use them to solve the problem.

GOALS

There are three main goals of the program:

1. Improving children's problem-solving abilities
2. Enriching children's mathematics abilities by teaching them problem-solving strategies
3. Assisting children in creating a "memory bank" of types of problems, solutions, and methods for arriving at solutions

Other objectives include:

1. Exercising children's high-level thinking skills by challenging them with problems that require the use of analysis, synthesis, and evaluation
2. Fostering children's sense of self-confidence in solving problems
3. Extending children's perseverance in attempting to solve challenging problems
4. Providing experiences that may serve to increase children's interest in mathematics

PROGRAM CONTENT

The problems in this book are designed to provide children with experience in choosing and using problem-solving strategies. The problems can't be solved by simply using an operation such as addition, subtraction, multiplication, or division; instead, children must use one or more of the following strategies, which have been adapted from the problem-solving strategies identified by the eminent mathematician George Polya:

Restate the problem in your own words.
Find the information given in the problem.

Identify what the problem is asking for.
Identify the conditions of the solution.
Use all given and implied information.
Solve a simpler problem.
Make a chart.
Draw a diagram.
Use trial and error.
Solve part of the problem.
Search for a pattern.
Work backward.
Consider other problems you've solved that might offer a solution.

The hints that accompany the problems include one or more of these strategies. Generally, if there is more than one hint with a problem, the hints build on one another to help children arrive at a solution. Occasionally, however, a hint suggests a strategy that will enable children to use an alternate method of arriving at a solution than the hint(s) before it pointed toward. As such, not every hint in every question is necessary to work through the problem (although often they are). That said, there are multiple ways of solving most of the problems beyond those laid out through the hints, and as children progress through the book, they should be encouraged to become less reliant on the hints, deciding on their own which strategies might work to solve the problems in a way that works best for them.

Some of the problems include one or more thinking extensions. There is a tendency, even among the best problem solvers, to consider a solution finished once they have obtained an answer. However, problem solvers lose a potentially valuable opportunity by stopping there. The thinking extensions can serve to help children consider what they have learned by applying it to similar problems or by using it in other contexts, encouraging them to play with the knowledge, to manipulate it, and thus to solidify it in their memory banks.

SETTING THE TONE

The learning atmosphere is one of the most important aspects affecting the success of a problem-solving program, and it depends largely on the actions and the attitude of the instructor. A supportive atmosphere is essential, especially when it comes to children taking intellectual risks. Compliment children for playing hunches, and encourage others to be supportive. Your consistent enthusiasm and praise can make an enormous difference.

Take actions to ensure that children have plenty of successes during the early part of the program. As children experience success resulting from their persistent efforts, both their willingness to attempt problems and their perseverance in finding solutions will increase.

Encourage collaboration among groups of children, if possible, and allow them plenty of time to solve problems and respond to questions. Persistence is more important than how quickly someone solves a problem. Similarly, concentrate on the *process* of solving problems. Emphasize using and choosing problem-solving strategies, and give less attention to the answer. When reviewing solutions, don't ask, "What is the answer?"; ask, "How did you solve it?"

USING THE PROGRAM

Assign the Problems in Sequence

This program was designed with the idea that children will work the problems in order. Generally, less difficult problems appear earlier in the book, and more difficult problems appear later. Also, children's understanding of a problem appearing later in the book may be contingent on their familiarity with an earlier problem. Children will be better prepared to work each problem if they have worked the problems that come before it.

A good method of implementing this program is to assign children one or two problems at the beginning of a week. Because the problems can be challenging, many children will not be able to solve a problem during one class period. Instead, they may need to spend time understanding a problem, investigating possible approaches to its solution, reflecting on how to utilize hints, debugging unsuccessful solutions, and satisfying themselves that a proposed solution is correct. To accomplish this, they should be allowed to work on an assigned problem over several days. Instructors can then review the solution with the children at the end of the week or once the children have indicated that they have completed the problem.

Introducing the Problem

To introduce a problem, read the problem, or ask a child to read it. Make sure the children understand the problem statement. If necessary, you may want to suggest strategy-directed hints for understanding the problem, such as:

> *Restate the problem in your own words.*
> *Find the information given in the problem.*
> *Identify what the problem is asking for.*
> *Identify the conditions of the solution.*

After the children have participated in the program long enough to become familiar with the problem-solving strategies, you can begin encouraging them to use strategies other than those suggested in the hints, reminding them that there are often multiple ways to solve problems and that no one way is the best way for everyone. The objective is for the children to develop a variety of problem-solving strategies.

1. Number Puzzler

Write a math problem that uses only the digit 9 four times and equals 100. Can you find 10 different solutions?

Hint 1

Use trial and error and mathematical operations you know to *solve a simpler problem*. Make three 9s equal 10.

We can do this with the following equation (among other possibilities): $\frac{9}{9} + 9 = 10$.

Hint 2

Solve a simpler problem. Make two 9s equal 1.

Here's one possibility: $9 \div 9 = 1$.

Answer

Here are some of the possible solutions:

1) $99\frac{9}{9}$
2) $99 + \frac{9}{9}$
3) $99 \div .99$
4) $(9 \text{ x } 9) \div (.9 \text{ x } .9)$
5) $9 \text{ x } 9 \div .9 \div .9$
6) $9 + (9 \text{ x } 9) \div .9$
7) $99 + (.9 \div .9)$
8) $\frac{9}{.9} \text{ x } \frac{9}{.9}$
9) $99 + (\sqrt{9} \div \sqrt{9})$
10) $99 + (\sqrt{.9} \div \sqrt{.9})$

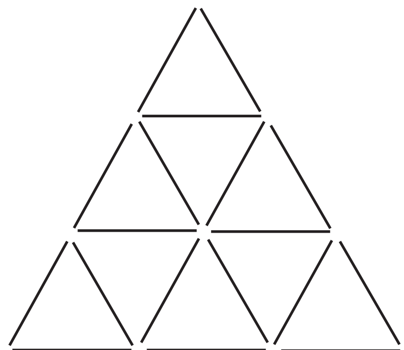
14. Toothpick Geometry

Let's do some toothpick geometry. Solve each of the following problems using the figure below:

a. Make six four-sided figures of the same size and shape by moving six of the toothpicks.

b. Remove five toothpicks to form five triangles.

c. Remove six toothpicks to form five triangles.



Hint 1

Solve part of the problem. What sorts of four-sided figures can you produce by moving or removing toothpicks?

Hint 2

Work backward.

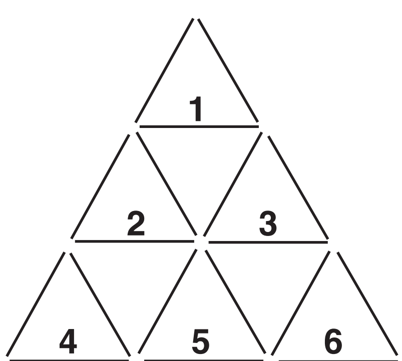
Hint 3

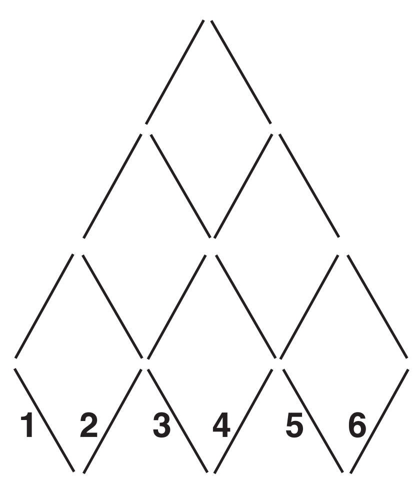
Use trial and error.

Answer

Some of the many possible solutions follow.

a.





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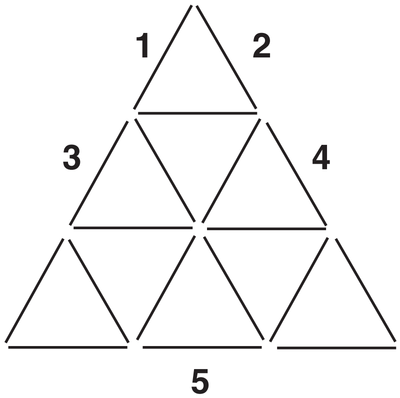
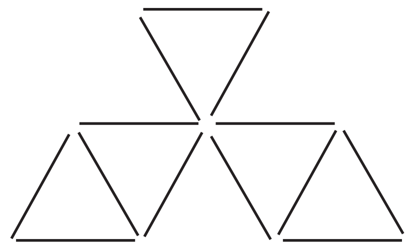
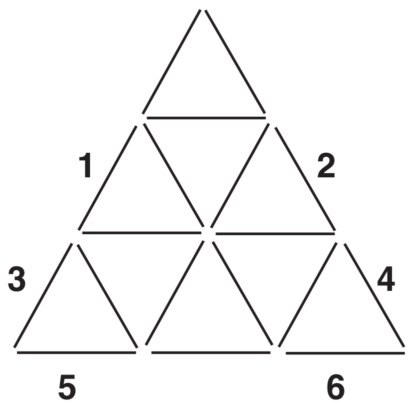
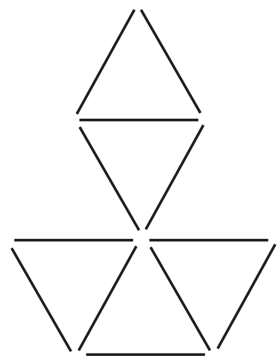




c.





25. Hop to It

This year the members of a local agricultural club plan to entertain the preschool children in their community with an animal race. They decide to mark a straight racecourse 100 feet in length. The winner will be the first animal to run to the end of the course and back.

The racing dog makes three-foot jumps, the goat leaps eight feet per jump, and the rabbit jumps two feet at a time. The rabbit, however, can jump three times each time the dog jumps twice and four times each time the goat jumps once.

The members of the agricultural club know which animal will win, but they plan to ask their young friends whether they can pick the winner before the race. Which animal will win? Which will finish second?

Hint 1

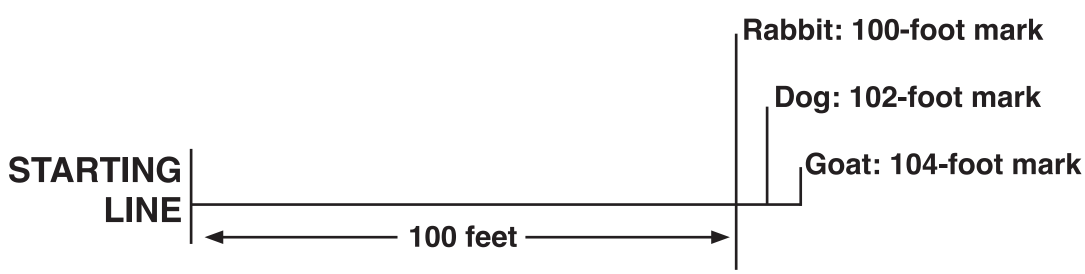
Solve part of the problem. How many jumps will it take the dog to reach the 100-foot mark?

If the dog jumps three feet at a time, then to reach 100 feet, it will need to jump $100 \div 3 = 33\frac{1}{3}$ times. But because it can't jump one-third of a time, it must jump 34 times to get to the 100-foot mark.

Hint 2

Draw a diagram of the racecourse and the jumps of the animals.

If the dog jumps 34 times, then it will jump 34 x 3 = 102 feet to cross the 100-foot mark. Similarly, the goat, which jumps eight feet at a time, will jump $100 \div 8 = 12\frac{1}{2}$ times, which we'll round to 13 jumps, and 13 x 8 = 104 feet. The rabbit, which jumps two feet at a time, will jump 100 ÷ 2 = 50 jumps, and 50 x 2 = 100 feet exactly.



Hint 3

Use all given and implied information. How far must the dog travel to complete the entire race?

The dog may take 34 jumps to get to the 100-foot mark, but it still has to return to the finish line, so it must jump an additional 34 times to get back, for a total of 68 jumps. And because the dog must jump to the 102-foot mark before starting back, it must travel a total of 204 feet to complete the racecourse. Similarly, the goat will take 13 jumps down and 13 jumps back, for a total of 26 jumps, which means that it will go 208 feet to complete the course. The rabbit will take 50 jumps each way, for a total of 100 jumps, traveling 200 feet.

Answer

Because the rabbit makes three jumps for every two that the dog makes, then the dog will jump 2 times for every 3 rabbit jumps, or $2/3$ of the 100 jumps that the rabbit will make to finish the race. This means that the dog will have jumped only $66\frac{2}{3}$ times when the rabbit crosses the finish line. But we know that the dog needs to jump 68 times to cross the finish line, so the dog will still be jumping when the rabbit has finished.

Similarly, because the rabbit makes four jumps for every one that the goat makes, then the goat will jump 1 time for every 4 rabbit jumps, or $1/4$ of the 100 jumps that the rabbit will make to finish the race. This means that the goat will have jumped only 25 times when the rabbit crosses the finish line. But the goat must jump 26 times to cross the finish line, so the goat, too, will still be jumping when the rabbit has finished.

Finally, we know that the dog is $2/3$ as fast as the rabbit, and the goat is $1/4$ as fast. We'll give those fractions common denominators to compare them better. The dog is $8/12$ as fast as the rabbit, and the goat is only $3/12$ as fast. So the dog will jump eight times for every three that the goat jumps. If the dog completes the racecourse in 68 jumps, then we can calculate $68 \div 8 = 8.5$, and the goat will have jumped only $8.5 \times 3 = 25\frac{1}{2}$ times, still not having crossed the finish line before the dog. Therefore, the rabbit will win the race, and the dog will finish second.