Problemoids

Grade 6
Math Mentor

Revised Edition

Bill McCandliss
Albert Watson

Royal Fireworks Press
Unionville, New York

Introduction

Problemoids is a problem-solving program designed to meet the needs of intellectually gifted children, as well as those children who would benefit from an enriched mathematics program that is more advanced than that provided in the standard curriculum. It engages children in high-level thinking about challenging problems and provides a stimulating opportunity for them to develop and increase their repertoire of problem-solving strategies.

Problemoids Level 6 contains 50 problems, which generally increase in difficulty as children progress through the book. The problems differ from those in a typical curriculum in that solving them requires children to use the full spectrum of thinking skills in Bloom's Taxonomy, with special emphasis on high-level skills such as analysis, synthesis, and evaluation. The topics covered in the program extend, enrich, and reinforce topics typically covered in a sixth-grade mathematics curriculum: sets, numbers and numeration, operations, algebra, geometry, measurement, and probability and statistics.

As many as four strategy-directed hints accompany each problem to assist children in solving the problem and in learning problem-solving strategies. The solutions emphasize those strategies by illustrating how to use them to solve the problem.

GOALS

There are three main goals of the program:

1. Improving children's problem-solving abilities
2. Enriching children's mathematics abilities by teaching them problem-solving strategies
3. Assisting children in creating a "memory bank" of types of problems, solutions, and methods for arriving at solutions

Other objectives include:

1. Exercising children's high-level thinking skills by challenging them with problems that require the use of analysis, synthesis, and evaluation
2. Fostering children's sense of self-confidence in solving problems
3. Extending children's perseverance in attempting to solve challenging problems
4. Providing experiences that may serve to increase children's interest in mathematics

PROGRAM CONTENT

The problems in this book are designed to provide children with experience in choosing and using problem-solving strategies. The problems can't be solved by simply using an operation such as addition, subtraction, multiplication, or division; instead, children must use one or more of the following strategies, which have been adapted from the problem-solving strategies identified by the eminent mathematician George Polya:

Restate the problem in your own words.
Find the information given in the problem.

Identify what the problem is asking for.
Identify the conditions of the solution.
Use all given and implied information.
Solve a simpler problem.
Make a chart.
Draw a diagram.
Use trial and error.
Solve part of the problem.
Search for a pattern.
Introduce an element.
Work backward.
Consider other problems you've solved that might offer a solution.

The hints that accompany the problems include one or more of these strategies. Generally, if there is more than one hint with a problem, the hints build on one another to help children arrive at a solution. Occasionally, however, a hint suggests a strategy that will enable children to use an alternate method of arriving at a solution than the hint(s) before it pointed toward. As such, not every hint in every question is necessary to work through the problem (although often they are). That said, there are multiple ways of solving most of the problems beyond those laid out through the hints, and as children progress through the book, they should be encouraged to become less reliant on the hints, deciding on their own which strategies might work to solve the problems in a way that works best for them.

Some of the problems include one or more thinking extensions. There is a tendency, even among the best problem solvers, to consider a solution finished once they have obtained an answer. However, problem solvers lose a potentially valuable opportunity by stopping there. The thinking extensions can serve to help children consider what they have learned by applying it to similar problems or by using it in other contexts, encouraging them to play with the knowledge, to manipulate it, and thus to solidify it in their memory banks.

SETTING THE TONE

The learning atmosphere is one of the most important aspects affecting the success of a problem-solving program, and it depends largely on the actions and the attitude of the instructor. A supportive atmosphere is essential, especially when it comes to children taking intellectual risks. Compliment children for playing hunches, and encourage others to be supportive. Your consistent enthusiasm and praise can make an enormous difference.

Take actions to ensure that children have plenty of successes during the early part of the program. As children experience success resulting from their persistent efforts, both their willingness to attempt problems and their perseverance in finding solutions will increase.

Encourage collaboration among groups of children, if possible, and allow them plenty of time to solve problems and respond to questions. Persistence is more important than how quickly someone solves a problem. Similarly, concentrate on the *process* of solving problems. Emphasize using and choosing problem-solving strategies, and give less attention to the answer. When reviewing solutions, don't ask, "What is the answer?"; ask, "How did you solve it?"

USING THE PROGRAM

Assign the Problems in Sequence

This program was designed with the idea that children will work the problems in order. Generally, less difficult problems appear earlier in the book, and more difficult problems appear later. Also, children's understanding of a problem appearing later in the book may be contingent on their familiarity with an earlier problem. Children will be better prepared to work each problem if they have worked the problems that come before it.

A good method of implementing this program is to assign children one or two problems at the beginning of a week. Because the problems can be challenging, many children will not be able to solve a problem during one class period. Instead, they may need to spend time understanding a problem, investigating possible approaches to its solution, reflecting on how to utilize hints, debugging unsuccessful solutions, and satisfying themselves that a proposed solution is correct. To accomplish this, they should be allowed to work on an assigned problem over several days. Instructors can then review the solution with the children at the end of the week or once the children have indicated that they have completed the problem.

Introducing the Problem

To introduce a problem, read the problem, or ask a child to read it. Make sure the children understand the problem statement. If necessary, you may want to suggest strategy-directed hints for understanding the problem, such as:

Restate the problem in your own words.
Find the information given in the problem.
Identify what the problem is asking for.
Identify the conditions of the solution.

After the children have participated in the program long enough to become familiar with the problem-solving strategies, you can begin encouraging them to use strategies other than those suggested in the hints, reminding them that there are often multiple ways to solve problems and that no one way is the best way for everyone. The objective is for the children to develop a variety of problem-solving strategies.

1. Number Factory

Can you solve the following math puzzles? Can you find more than one way to solve some of them?

a. Write a math problem that uses only the digit 8 three times and equals 9.

b. Write a math problem that uses only the digit 4 three times and equals 11.

c. Write a math problem that uses only the digit 9 three times and equals 78.

d. Write a math problem that uses only the digit 3 three times and equals 24.

e. Write a math problem that uses only the digit 5 three times and equals 5.

Hint 1

Use trial and error and mathematical operations you know to *solve a simpler problem*. Make two 8s equal 1.

We can do this with the following equation (among other possibilities): $8 \div 8 = 1$.

Hint 2

Solve a simpler problem. Make two 9s equal 6.

Here's one possibility: $\sqrt{9} + \sqrt{9} = 6$.

Answer

Here are some of the possible solutions:

a. $8 + {}^{8}\!/_{8} = 9$

b. $44 \div 4 = 11$

c. $(9 \times 9) - \sqrt{9} = 78$

d. $3^3 - 3 = 24$

e. $5({}^{5}\!/_{5}) = 5$

$(5 - 5) + 5 = 5$

$5^{(.5 + .5)} = 5$

$(.5 + .5)5 = 5$

$(.5 - .5) + 5 = 5$

$({}^{.5}\!/_{.5})\ 5 = 5$

$(\sqrt{5} - \sqrt{5}) + 5 = 5$

16. All for One

Jayden wants to weigh himself and his two younger sisters on the scale at the county fair, but they only have enough money left to use the weight scale once. Jayden knows that he weighs 30 pounds more than the weight of both of his sisters together and that his baby sister weighs 80% less than her older sister. So Jayden confidently tells his sisters, "Get on the scale with me, and I'll calculate our individual weights after I know our combined weight."

The scale shows that the siblings weigh 210 pounds together. How much does each person weigh?

Hint 1

Solve a simpler problem. If a dog and a cat weigh 12 pounds together, and the cat weighs 50% less than the dog, how much does each animal weigh?

Because the cat weighs 50% less than the dog, the dog must weigh twice as much as the cat. This means that the weight of both together must be three times the weight of the cat. Thus, the weight of the cat is 12 pounds ÷ 3 = 4 pounds, and the weight of the dog is 2 x 4 pounds = 8 pounds.

Hint 2

Find the information given in the problem. Use the information to construct some equations.

We know that Jayden's weight minus 30 pounds equals the weight of his two sisters. We also know that Jayden's weight plus the weight of his two sisters equals 210 pounds.

Let's let j equal Jayden's weight and s equal the weight of his two sisters.

$$j - 30 = s$$
$$j + s = 210$$
$$j + (j - 30) = 210$$
$$2j = 210 + 30 = 240$$
$$j = 120$$

So Jayden must weigh 120 pounds, and his sisters must weigh 210 – 120 = 90 pounds together.

Hint 3

Use all given and implied information to find what percentage of the older sister's weight equals the weight of both sisters together.

The weight of the baby sister is 80% less than that of the older sister. This implies that 20% of the older sister's weight equals the weight of the baby sister, which means that 120% of the older sister's weight equals 90 pounds.

Answer

Let's let s_1 equal the older sister's weight and s_2 equal the baby sister's weight. We already know that the older sister weighs 120% of 90 pounds and that the baby sister weighs 20% of the older sister.

$120\% \times s_1 = 90$

$s_1 = 90 \div 120\%$

$s_1 = 75$

$s_2 = 20\% \times 75$

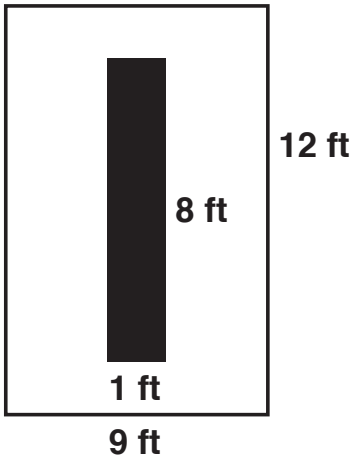
$s_2 = 15$

Therefore, Jayden weighs 120 pounds, his older sister weighs 75 pounds, and his baby sister weighs 15 pounds.

46. Cutting a Rug

The principal of a small school needed a rug to fit into a playroom that measured 10 feet by 10 feet. He asked the school parents' association for the donation of a second-hand rug for the room until the school could purchase one. One of the parents offered to donate a rug that measured 9 feet by 12 feet with a hole in the middle that measured 1 foot by 8 feet. The principal didn't think that the rug would fit, and he was about to decline the donation when a student standing nearby suggested, "We can cut the rug into two pieces and sew them together to made it a perfect fit for our little playroom."

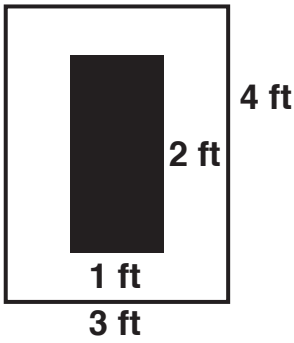
How was the student planning to cut the rug?

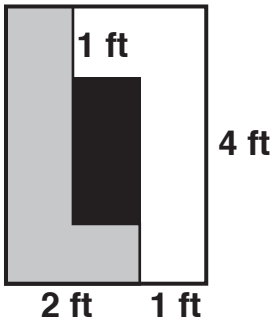


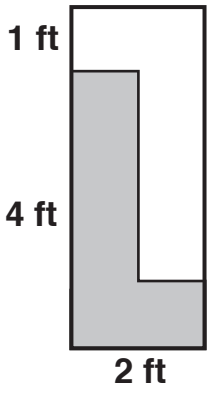
Hint 1

Solve a simpler problem. Suppose the rug is 3 feet by 4 feet and has a 1-foot by 2-foot hole in it. How can the school cut the rug into two pieces that can be sewn together into a rug that measures 2 feet by 5 feet? *Draw a diagram* to help you visualize the solution.

Let's draw diagrams of the rug so we can see how to make it work.



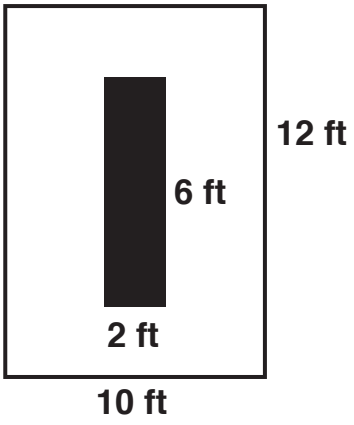


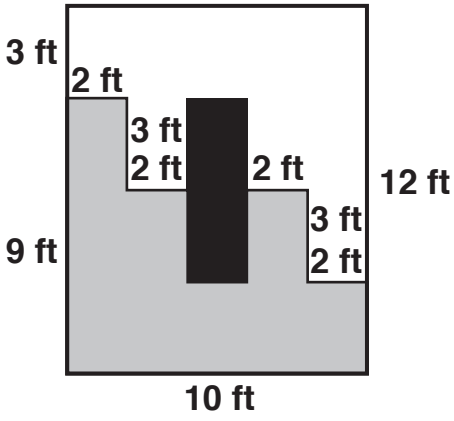


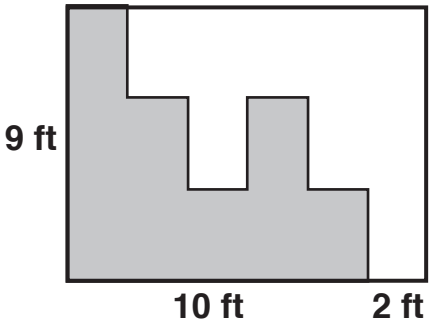
Hint 2

Solve a simpler problem. Can you cut a 10-foot by 12-foot rug with a 2-foot by 6-foot hole in the middle into two pieces that can be sewn together to make a 12-foot by 9-foot rug?

Here again, we'll draw diagrams to show how a rug this size could be divided into two pieces that can be put back together into the desired shape.







Answer

Now that we can see that it takes a "stair-step" approach to cutting the rug to make the pieces fit together in the way we want, we are in a better position to cut the 9 x 12 rug and put it back together to form a 10 x 10 rug.