Classical Logic

An Illustrated Introduction

Volume I

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Contents

Introduction......1

Unit One: Claims and Arguments 5

Chapter 1: Logic and Its Laws 5

Chapter 2: Claims......11

Chapter 3: Claims in Relationships 14

Chapter 4: Arguments 18

Chapter 5: Argument Types 25

Chapter 6: Validity and Soundness...... 29

Unit Two: Logical Code Language 33

Chapter 7: Code for Categories...... 33

Chapter 8: Code for Individuals 43

Chapter 9: Harder Translations 49

Chapter 10: The Square of Opposition...... 56

Unit Three: Categorical Syllogisms 68

Chapter 11: The Structure of Syllogisms...... 68

Chapter 12: Rules for Validity: Affirmation and Negation 74

Chapter 13: Rules for Validity: Distribution...... 78

Unit Four: Shortcuts...... 86

Chapter 14: Schemas 86

Chapter 15: Mood and Figure...... 89

Chapter 16: The Medieval Cheat Sheet 94

Chapter 17: A Problem with Existence......100

Conclusion: Beyond Syllogistic Logic......103

Appendix: Longer Arguments from Lewis Carroll......104

Answer Key 107

Introduction

This logic book is written in the spirit of Lewis Carroll. It aims to teach mastery of a seemingly forbidding subject and to make it enjoyable for both young adults and those who are merely children at heart.

Lewis Carroll (1832-1898) was a unique combination of logician, mathematician, and children's author. His real name was Charles Dodgson, and his true profession was mathematics. His beloved children's books—*Alice in Wonderland*, *Through the Looking Glass*, *Sylvie and Bruno*—are full of jokes and allusions enjoyed by school children and professional logicians alike. Carroll even published a little book on symbolic logic for the young teens of his day. The book "gamified" logic in the late 1800s in ways that logic texts are only beginning to do today.

Formal logic has now developed well beyond Carroll's methods. Carroll himself was part of a movement to revise the old classical syllogistic system that had developed over nearly 2,000 years. In the process of revision, the classical system was eventually overhauled into the more powerful logics in use today. But the concepts in classical logic—and the spirit of Carroll's teaching methods—are as relevant as ever.

As Carroll saw it, classical logic is the best game for building what everyone now calls "transferable skills." Carroll told his young readers that logic is more useful than chess or backgammon, since its rules apply universally and since the game can be played anywhere. Logic is thus the game that trains us to play all games. Carroll also pointed out that logic is good for "mental health," meaning *mental* in its original sense of "whatever pertains to being smart." Logic makes us smarter by giving us tools for forming clear ideas and arranging them in the inner landscape of our knowledge. In addition (and not insignificantly), logic is powerful at detecting and dismantling fallacies.

This book thus invokes Carroll's spirit both in its aims and its methods. The subject is syllogistic logic as it developed from Aristotle to the late 1800s. The examples and exercises are supplemented by content from Carroll's logic book and his fictional writings. And the pictures ("what is the use of a book without pictures?"*) are taken from or inspired by early editions of Carroll's children's books.

What advice should we give to students beginning logic? None better than Carroll himself gave. Written more than a hundred years ago in his introduction to *Symbolic Logic*, Carroll told the young at heart to be diligent—and also to enjoy the game.

*As Alice asks herself in the first chapter of *Alice in Wonderland*

Lewis Carroll's Advice to Logic Students

Symbolic Logic **(1896)**

The Learner, who wishes to try the question fairly, whether this little book does, or does not, supply the materials for a most interesting mental recreation, is earnestly advised to adopt the following Rules: —

(1) Begin at the beginning, and do not allow yourself to gratify a mere idle curiosity by dipping into the book, here and there. This would very likely lead to your throwing it aside, with the remark "This is much too hard for me!", and thus losing the chance of adding a very large item to your stock of mental delights. This Rule (of not dipping) is very desirable with other kinds of books — such as novels, for instance, where you may easily spoil much of the enjoyment you would otherwise get from the story.... This, I say, is just permissible with a novel...but, with a scientific book, it is sheer insanity: you will find the latter part hopelessly unintelligible, if you read it before reaching it in regular course.

(2) Don't begin any fresh Chapter, or Section, until you are certain that you thoroughly understand the whole book up to that point, and that you have worked, correctly, most if not all of the examples which have been set. So long as you are conscious that all the land you have passed through is absolutely conquered, and that you are leaving no unsolved difficulties behind you, which will be sure to turn up again later on, your triumphal progress will be easy and delightful. Otherwise, you will find your state of puzzlement get worse and worse as you proceed, till you give up the whole thing in utter disgust.

(3) When you come to any passage you don't understand, read it again: if you still don't understand it, read it again: if you fail, even after three readings, very likely your brain is getting a little tired. In that case, put the book away, and take to other occupations, and next day, when you come to it fresh, you will very likely find that it is quite easy.

(4) If possible, find some genial friend, who will read the book along with you, and will talk over the difficulties with you. Talking is a wonderful smoother-over of difficulties. When I come upon anything — in Logic or in any other hard subject — that entirely puzzles me, I find it a capital plan to talk it over, aloud, even when I am all alone. One can explain things so clearly to one's self! And then, you know, one is so patient with one's self: one never gets irritated at one's own stupidity!

If, dear Reader, you will faithfully observe these Rules, and so give my little book a really fair trial, I promise you, most confidently, that you will find Symbolic Logic to be one of the most, if not the most, fascinating of mental recreations....

Mental recreation is a thing that we all of us need for our mental health; and you may get much healthy enjoyment, no doubt, from Games, such as Back-gammon, Chess, and the new Game "Halma". But, after all, when you have made yourself a first-rate player at any one of these Games, you have nothing real to show for it, as a result! You enjoyed the Game, and the victory, no doubt, at the time: but you have no result that you can treasure up and get real good out of. And, all the while, you have been leaving unexplored a perfect mine of wealth. Once master the machinery of Symbolic Logic, and you have a mental occupation always at hand, of absorbing interest, and one that will be of real use to you in any subject you may take up. It will give you clearness of thought — the ability to see your way through a puzzle — the habit of arranging your ideas in an orderly and get-at-able form — and, more valuable than all, the power to detect fallacies, and to tear to pieces the flimsy illogical arguments, which you will so continually encounter in books, in newspapers, in speeches, and even in sermons, and which so easily delude those who have never taken the trouble to master this fascinating Art. Try it. That is all I ask of you!

L.C.
29, BEDFORD STREET, STRAND.
February 21, 1896.

Unit One: Claims and Arguments

Chapter 1: Logic and Its Laws

On the way to fight jabberwocks, we often get lost in tulgey woods. What can help us find our way? Logic.

The world, as you may already have discovered, is a tulgey wood full of worse things than tumtum trees. There are swamps with bottomless mudholes and few footholds. But there are also a few tree stumps to help us across. For instance, look at the tree stumps that follow. Which conclusions are correct?

1. No one who gets lost in a tulgey wood is safe from jabberwocks.
   You're safe from jabberwocks.
   Therefore:

   (a) You're lost in a tulgey wood.
   (b) You're not lost in a tulgey wood.
   (c) The tulgey wood is full of jabberwocks.
   (d) None of the above.

2. Anyone surrounded by tumtum trees is lost in a swamp.
   You're not surrounded by tumtum trees.
   Therefore:

   (a) You're lost in a swamp.
   (b) You're not lost in a swamp.
   (c) There are no swamps.
   (d) None of the above.

3. Anyone who has the Lifeline of Logic can find the way through a tulgey wood.
   You don't yet have the Lifeline of Logic.
   Therefore:

   (a) You can't yet find your way through a tulgey wood.
   (b) You can still find your way through a tulgey wood.
   (c) There are no jabberwocks in the tulgey wood.
   (d) None of the above.

4. Aristotle gave us the Lifeline of Logic.
   Anyone who gave us the Lifeline of Logic was either a philosopher or a wizard king.
   Therefore:

   (a) Aristotle was a philosopher.
   (b) Aristotle was a wizard king.
   (c) At least one of the above.
   (d) None of the above.

5. Every philosopher is a fine logician.
   Every wizard king is a fine logician.
   Therefore:

   (a) Every philosopher is a wizard king.
   (b) Every wizard king is a philosopher.
   (c) Both of the above.
   (d) None of the above.

Answer Key: How many did you get right?
1b, 2d, 3d, 4c, 5d

Why do we get lost in tulgey woods? Our minds are adventurers, always jumping beyond what we know and into the great beyond. The process starts with something we have been told—like the stump of a tumtum tree giving us a foothold. The act of jumping to something we *haven't* been told is called *inference*, or *reaching a conclusion*, or *seeing what follows from something else*.

Inferring may sound like a good idea, but it's as tricky as jumping from stump to stump in a swamp. We can make mistakes. The tree stumps must be arranged in the right way.

Logic is the study of how to move from claim to claim rightly and safely. More precisely, **logic is the study of how to conclude something from something else**. Logic gives us the most basic principles of reasoning. These principles tell us how to conclude some things reasonably and reliably. Logic is also the study of how our reasoning can go wrong. We make mistakes and commit fallacies. Logic helps us correct ourselves.

The word *logic* comes from the Greek word *logos*, meaning "reason." Reason is our guide through the great muddy swamplands of human thought. In those swamplands there may be jabberwocks. Logic is one of our most effective ways of dealing with them.

The Laws of Logic

For dealing with jabberwocks, good rules are essential. Logic has three famously good rules:

1. **The Law of Identity**: A thing is itself.
2. **The Law of Non-Contradiction**: A thing cannot be something and its opposite in the same respect at the same time.
3. **The Law of the Excluded Middle**: Any statement is either true or false. There is no "middle option."

These rules are good in a peculiar sense. It's not merely because a wizard-king sat down and thought them up. It's not because they work in most cases, like handgrips for tree-climbing and vorpal swords for jabberwock-fighting. Rather, the rules are good because without them, all logic is impossible.

Let's take the following example from math: 1+1 = 2. This is a classic inference. It's also a strange one. The best logicians and mathematicians don't quite understand how we make this inference. But one thing is clear: We can't make it at all unless the three laws of logic are true.

1. According to the Law of Identity, the number 1 is identical to itself.
2. According to the Law of Non-Contradiction, the number 1 cannot be odd *and* non-odd (even) at the same time. It also cannot be prime *and* non-prime, or positive *and* negative. (Imagine the chaos!)
3. According to the Law of the Excluded Middle, the statement "1+1=2" must be either true or false. There is no third option.

How do we know we can trust these laws? One way is to try to doubt them. Can we deny these laws without using the laws themselves? For instance, let's say that a neighborhood jabberwock puts up a cardboard sign. It says:

The laws of logic are false.

How can we reason with this jabberwock?

Law 1. First, we might ask the jabberwock (politely) if he thinks a law of logic is a law of logic. If he says yes, then he has affirmed Law 1, the Law of Identity. If he says no, then his sign isn't itself. Its topic is also not its topic. He (and we) don't know what the sign actually says. "The laws of logic are false" could be the same as "The dormouse's grandmother is sick" or "The duchess does not like tea." In any event, the sign wouldn't mean that the laws of logic are false because the sign wouldn't mean the laws of logic at all.

Law 2. Second, we might ask the jabberwock if his cardboard sign is a joke. If he insists that it's *not* a joke—He is quite serious!—then he has affirmed Law 2, the Law of Non-Contradiction. He assumes that his sign can't be *both* a joke *and* its opposite at the same time. On the other hand, the jabberwock might answer yes: his sign is both a joke and not a joke. If that's true, what relief! The sign is a joke, and we don't have to take it seriously. The jabberwock has again failed to deny (seriously) a law of logic, despite his best efforts.

Law 3. Third, suppose we ask the jabberwock about the excluded middle. Is his sign either true or false? If the jabberwock answers *true and not false*, then he has affirmed Law 3. If he answers *false and not true*, then he has still affirmed Law 3. If he answers *both true and false*, then he tries to deny Law 3. However, in doing so, he admits that his claim is false. Our work is done. If he answers *neither true nor false*, then he also tries to deny Law 3. But in doing so, he admits that his claim isn't true. Again our work is done. In every case, the jabberwock can't actually deny Law 3, no matter how hard he tries!

The laws of logic are hard to get around. If we try to prove them, we end up assuming them. If we try to deny them, we end up affirming them. Most logicians think they're inescapable. If anything in human reasoning is trustworthy, those laws are trustworthy.

Limits of the Laws

Before we go gallumphing into a tulgey wood with the laws of logic, we should know their limits. These laws only work on *well-formed claims*. They don't work on nonsense or other sentences that aren't real claims.

Do these sentences obey the laws of logic?
(Read the next few pages to find out!)

1. The mome raths outgrabe.
2. You have stopped celebrating your un-birthdays.
3. The March Hare and the Hatter were having tea.
4. This sentence is false.

What follows are four different types of sentences that sometimes pose problems for the laws of logic. In each case, most logicians think that the problem is with the sentence and not with the laws. As you read each type, ask yourself, "What is the real problem here?"

1. **Nonsense sentences.** "The mome raths outgrabe" is not clearly true or false. That's because *mome raths* and *outgrabe* don't have clear meanings. Because they don't represent definite things or ideas, there's no truth about whether they're the same or not. So has Law 3 been broken? Most logicians say no. Law 3 presumes that a claim says something clear about an identifiable subject. If the claim doesn't do that, then Law 3 simply doesn't apply.

It works similarly with sentences that are questions and commands. The sentence "Hast thou slain the jabberwock?" is not a claim, so it also can't be true or false. The command "Slay the jabberwock!" follows the same principle. Because these sentences are not claims, Law 3 doesn't apply.

2. **Loaded claims.** Suppose I remark to you one day, "You have stopped celebrating your un-birthdays!" If you are like most people, you have never celebrated an un-birthday. So is my remark true or false? If true, then you have stopped—which is false, since you never began. But if my remark is false, then you haven't stopped. That means you are still celebrating—which is false, since you aren't. So it looks as though my remark is neither true nor false. Have I finally broken Law 3?

   The good news is that most logicians say no. My slippery remark is simply not *well formed*. It smuggles in *two* claims and disguises them as one. Claim 1 is that you began celebrating un-birthdays, and Claim 2 is that you've stopped. Claim 2 can't be true or false without assuming Claim 1. And if Claim 1 is false, then Claim 2 makes no sense.

3. **Fictional sentences.** On the subject of un-birthdays, what about fictional sentences? For example, *Alice in Wonderland* contains sentences like this one:

   > "The March Hare and the Hatter were having tea."

   Is that sentence true or false? If there are no such characters in reality, we might think *false*. But there *are* such characters in the world of Wonderland. Because the sentence was written by Lewis Carroll, we might take his word for the truth. Is the sentence both true and false at once?

   Logicians have different answers. Some logicians think that this sentence breaks Law 3. However, most logicians think otherwise. Perhaps the sentence is not well formed, since it's not about the real world. Or perhaps it *is* a well-formed claim, just one about a fictional world. Relative to Wonderland, the claim might be genuinely true or false (but not both at the same time). Either way, most logicians don't think it breaks Rule 3.

The key to fictional worlds is to be very clear about them. Not everyone agrees that a March Hare ever had tea with a Mad Hatter. But everyone agrees that the following sentences are true:

> "In *Alice in Wonderland*, Lewis Carroll wrote that the March Hare and the Hatter were having tea."
>
> or
>
> "The March Hare and the Hatter were having tea in the fictional world of Wonderland."

If you are ever in doubt about a fictional sentence, you can simply translate it into a form that is true. A clear claim recognizes the setting of a fictional world. And within that setting, the claim does not break Law 3.

4. **Paradoxes of self-reference.** Finally, what about the most slippery claim of all? Logicians sometimes point to a famous sentence that seems to break Law 3 entirely. It looks like this:

   > "This sentence is false."

   What should we do with this sentence? If it's true, then it's false. But if it's false, then it's true. Does it break Law 3?

   This slippery claim is known as the "Liar Paradox." There's no easy solution. Some logicians say that Law 3 simply breaks; the claim is *neither* true *nor* false. These logicians admit that Law 3 may not be as universal as it looks.

   However, most logicians think differently. They say that the Liar Paradox isn't well formed. It refers to itself in a way that doesn't work. If a line of computer code referred to itself like this, it would break. Because the Liar Paradox is poorly formed, it breaks the laws of good writing, but it doesn't break Law 3.

All logicians agree that it's important for claims to be well formed. We can't apply the laws of logic if we only have nonsense or slippery claims. Questions, commands, and self-referring sentences are not well formed. But once our claims are in order, the laws will work.

There is much more to know about claims. Claims are the tree stumps and branches of arguments. But even with the laws on our side, arguments can be hard to follow. In the next chapter, we'll look at claims and their roles, both inside arguments and out.

Chapter 2: Claims

Imagine the Knave of Hearts being cross-examined at a trial. As far as I know, he never had this conversation, but in Wonderland, who can tell?

> King of Hearts: "Are you animal, vegetable, or mineral?"
> Knave: "No, so please Your Majesty—"
> Queen of Hearts: "Off with his head!"

Has anyone in this conversation made a claim?

Logicians are precise when they use the word *claim*. In logic, a **claim** is a sentence that is either true or false. It affirms or denies something. It doesn't ask questions or command anything. For example:

> A claim is *not* a question: "Did you steal the tarts?"
> A claim is *not* a command: "Hand over the tarts!"
> A claim is *not* an exclamation of emotion: "How wonderful—tarts!"

Instead, here are some claims. Each is either true or false:

> "The knave has stolen the tarts."
> "The queen wishes to have his head cut off."
> "I do not think that the queen is a nice person."

Sometimes we call claims "propositions." The word *proposition* comes from the Latin stems *pro* and *positio*, meaning "out-front position." Propositions set something out as being true, like a bakery setting out tarts and cakes in the front window. Propositions always want you to pick them up and swallow them down whole. But in reality, watch out—they can be either true or false.

Here is a table with a few more examples. Study them before trying Exercise 2.1, which follows.

Claim/Proposition	Not a Claim/Proposition
I'm late!	Am I late?
Jabberwocks eat tarts.	Watch out for jabberwocks eating tarts.
Tart-making requires time.	Oh, the time it takes to make tarts!
I am baking tarts.	Are you baking tarts?

Exercise 2.1

The quotations that follow are taken from the *Alice* stories. Which ones are propositions? (The answers to this exercise—and every exercise you'll encounter in these pages—are in the answer key in the back of this book.)

1. Alice was beginning to get very tired of sitting by her sister on the bank.
2. The Duchess! The Duchess! Oh my dear paws!
3. If he smiled much more, the ends of his mouth might meet behind.
4. Why, Mary Ann, what are you doing out here? Run home this moment, and fetch me a pair of gloves and a fan! Quick, now!
5. I shall be late!
6. The rule is, jam to-morrow and jam yesterday—but never jam to-day.
7. What a fight we might have for the crown!
8. Sometimes I've believed as many as six impossible things before breakfast.
9. Speak in French when you can't think of the English for a thing—turn out your toes as you walk—and remember who you are!
10. A "borogove" is a thin shabby-looking bird with its feathers sticking out all round.
11. Always speak the truth—think before you speak—and write it down afterwards.
12. Collar that Dormouse! Behead that Dormouse! Off with his whiskers!

Unusual Propositions

In tulgey woods—and in real life—sometimes propositions are hard to spot. Words may not *look* like claims, even if they are. Logic helps remove the disguises. Following are three types of utterances that can be rephrased as claims, given a little context and creativity.

Yes and No. What happens when we say *yes*, *no*, *of course*, *absolutely not*, or *never*? These words aren't complete sentences. However, they affirm or deny something, so they must have the weight of propositions. Their exact meaning depends on the context. If I simply shout "Yes!" right now, with no previous question, then my "yes" is not a claim. But if my "yes" is a response to the question "Do you like tarts?," then it is a claim. It's a quick way of saying, "I do like tarts!"

Negations are similar. You may ask me, "Did you steal the tarts?" I might answer "No" or "Never" or "Of course not, you scoundrel!" All of these denials amount to the same proposition: "I did not steal the tarts." So when you translate my speech into logic, you would phrase my denial as a claim.

Questions and commands. This chapter has already emphasized that questions and commands are not claims. But there's good news: They can be rephrased as claims, depending on context. Here we must be careful. When we change questions and commands into claims, they are not about their subject but about the speaker. For instance, instead of asking, "Did you steal the tarts?," I could say, "I want to know whether you stole the tarts." My question has now become a claim—not about you, but about my curiosity. Similarly, if I order you to "Hand over the tarts!," then my order can be rephrased as a claim: "I want you to hand over the tarts."

These claims, of course, can be true or false. If I don't care whether you stole the tarts, and if I don't want you to hand them over, then both claims are false. So these rephrased claims also obey the laws of logic.

In short, when we see questions and commands in daily life, we should not despair of making them logical. We may simply translate them into claims. And we can always do this with "yes" and "no" when we have enough context.

Exercise 2.2

Read the conversation that follows. Some of the lines are from Lewis Carroll's *Symbolic Logic*. Can you spot the propositions? If a line *isn't* a proposition, rewrite it as one. (You can use some imagination, but make sure you write a full sentence that is true or false.) The answer key includes Carroll's own suggestions for how to revise some of the propositions.

1. I see John.
2. Do you see John?
3. No.
4. Oh!
5. Fetch me that book.
6. Which book do you mean?
7. I want that book over there.
8. Never!