MATHEMATICAL HISTORIES, VOLUME 1

In Search of ANCIENT ALGORITHMS

ROBERT BLACK

Royal Fireworks Press
Unionville, New York

Also by Robert Black,
published by Royal Fireworks Press:
The Mathematical Lives series
The Mathematical Nights series
Liberty Girl
Unswept Graves
The Eyes of the Enemy



Copyright © 2026
Royal Fireworks Online Learning, Inc.
All Rights Reserved.

Royal Fireworks Press
P.O. Box 399
41 First Avenue
Unionville, NY 10988-0399
(845) 726-4444
fax: (845) 726-3824
email: mail@rfwp.com
website: rfwp.com



ISBN: 978-1-63856-309-9

Publisher: Dr. T.M. Kemnitz
Editor: Jennifer Ault
Book Designer: Kerri Ann Ruhl
Cover Design: Kerri Ann Ruhl & Christopher Tice

Printed and bound in Unionville, New York,
at the Royal Fireworks facility. 27o26

Table of Contents

Prologue 1

Part One 4

Chapter 𒌋: The Land of the Mathematical King 6

Chapter 𒌋𒌋: From Counters to Cuneiform 10

Chapter 𒐕: "The entrance into the knowledge of all existing things and all obscure secrets" 16

Chapter 𒐖: Tadpoles, Lotus Blossoms, and an Astonished Man 21

Chapter 𒐗: The Land of the Gnomon and Compass 27

Chapter 𒐚: Bamboo Mathematics 31

Part Two 37

Chapter 𒐝: The First Computers 38

Chapter 𒐼: Going Negative 44

Chapter 𒐽: From the Pyramids to Your Smartphone 47

Chapter 𒑁: Units Only, Please 53

Chapter 𒑊: The Same but Different 59

Chapter 𒑋: Mothers and Sons 66

Chapter 𒑏: Who Needs Division Anyway? 71

Part Three 79

Chapter 𒑐: Finding Truth by Being False 80

Chapter 𒑑: When "Completing the Square" Was Meant Literally 87

Chapter 𒑒: The Hook, the Leg, and the Bowstring 94

Part Four 102

Chapter 𒑓: The First Greek Philosopher 104

Chapter 𒑔: A Fateful Visit 107

Chapter 𒑕: Beyond the Right Answers 112

Epilogue 117

Doing the Math 1: The Binary Multiplication Algorithm 118

Doing the Math 2: The Euclidean Algorithm, in Chinese 121

Doing the Math 3: The Method of False Position 123

Doing the Math 4: The "Akkadian Method," or "Completing the Square" 125

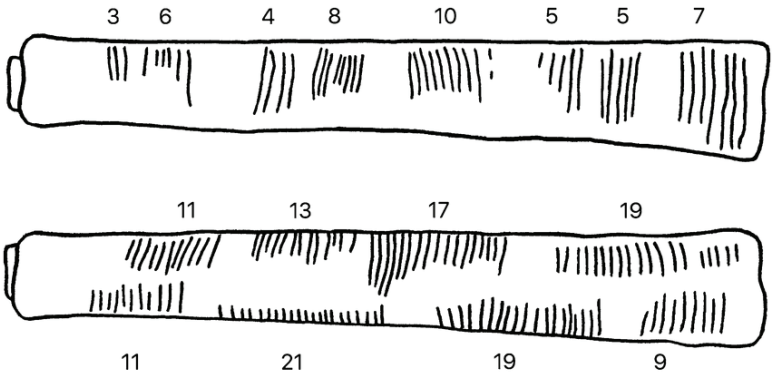
About the Author 128

PROLOGUE

Sometime in humanity's distant past, about 22,000 years ago, one of our prehistoric ancestors fashioned a useful little tool. Today, if you visit the Royal Belgian Institute of Natural Sciences in Brussels, you can see it for yourself. Found in 1950 by geologist Jean de Heinzelin de Braucourt near the village of Ishango in what is now the Democratic Republic of Congo, the tool is about the size of a half-used pencil. It was made from the leg bone of a baboon or a similar-sized mammal, with a piece of sharp quartz attached to one end. But that's not what makes it special. What earns it a place in the Royal Belgian Institute of Natural Sciences is what's been marked on it.



The entire length of the Ishango Bone, as it is called, is incised with three rows of notches, grouped together in an irregular pattern. On one row, there are three notches, followed by six, then four, eight, ten, five, five, and seven. On another, there are eleven notches, followed by thirteen, seventeen, and nineteen. And on the third, there are again eleven notches, followed by twenty-one, nineteen, and nine.



What do they mean? Do they mean anything at all? Examining the notches under a microscope reveals that they were made at different times. Was someone using the bone to keep count of something? Similar "tally sticks" have been found elsewhere around the world, some of them even older than the Ishango Bone. But some scientists, beginning with de Heinzelin himself, have claimed to see more advanced explanations. The first row of notches could suggest the practice of doubling—6 is 3 doubled, 8 is 4 doubled, and 10 is 5 doubled. The groupings in the second row are all of the prime numbers between 10 and 20. And the groupings in the third row suggest knowledge of a base-10 number system, since $11 = 10 + 1$, $21 = 20 + 1$, $19 = 20 - 1$, and $9 = 10 - 1$. Other researchers claim to have found a relationship between the groupings and the lunar calendar and conclude that the bone represents the lunar cycle over a six-month period.

Are any of these claims correct? Most historians and archaeologists believe that primitive humans didn't understand mathematical concepts like these. Could a tribe of nomads in central Africa really have known about them, thousands of years before anyone else did?

The answer is that we don't know. The Ishango Bone is unique. Nothing similar to it has ever been found—not even among other relics left by the same people. It exists in a vacuum, and a vacuum can be filled by anything. Imagine a group of archaeologists in the distant future finding a single lottery ticket or credit card. If they knew nothing else about our civilization, what might they imagine those numbers meant?

Human beings love patterns. Our survival as a species has depended in part on our ability to recognize them. They can help us predict what will happen in the future, or when things will happen. They can help us anticipate both danger and opportunity. And they can help us make copies of things that exist or make new things that never existed before. Even when our survival doesn't depend on it, we sometimes entertain ourselves by creating patterns out of numbers or shapes.

Thousands of years after the Ishango Bone was carved, but still in our own distant past, people learned how to grow their food instead of hunting and gathering all the time, and that enabled them to settle down and build the first communities. As they did, the patterns they needed to recognize and work with began to change. They no longer needed to survive in the wilderness, but civilization brought new demands and new challenges. Meeting those challenges required recognizing new patterns and learning how to use them. In time, the study and use of those patterns became its own discipline. Today, we call it mathematics.

Civilization took root in many different places around the world, and each one developed its own mathematical methods. On the surface, they all look different from one another, and they also look different from math as we know it today. But if you examine them more closely, you might be surprised by how much they have in common. There are many ways to describe a pattern and

many ways to solve a problem, but look beneath the surface, and you'll find that the math is always the same. By exploring the different paths taken to reach the same goals, we can learn something new about the nature of the patterns themselves. And who knows where that will lead next?

PART ONE

Before anything else, there was counting.

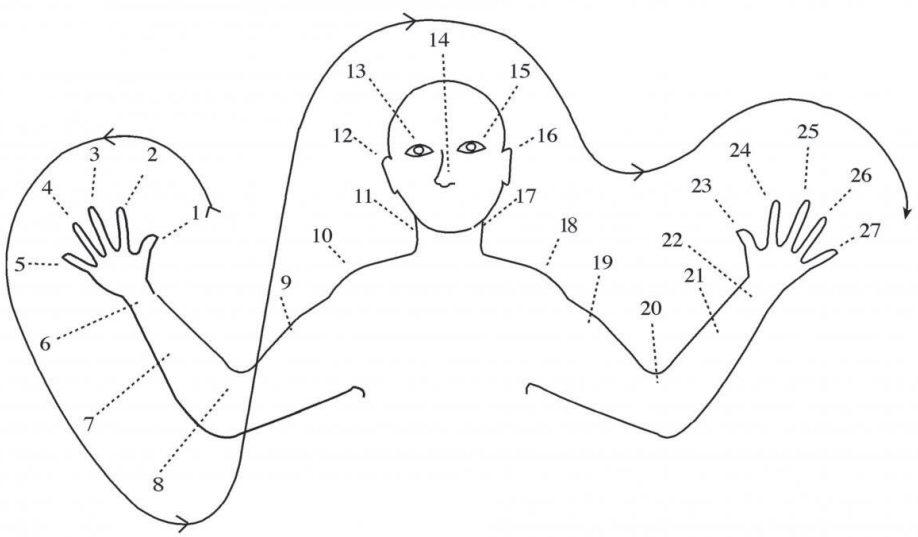
That was probably your first taste of mathematics. Before anything else, you learned how to count the number of fingers and toes you have, the number of toys you could play with, the number of bites of something you didn't like that you had to eat, and so on. It's the most basic mathematical property, and often the one people are most interested in.

Humanity's ancestors began that way, too. Tens of thousands of years ago, people began to recognize that things could be counted, that *quantity* was something they could see and name. Perhaps at first people understood only the numbers 1, 2, and possibly 3. Any amount beyond that was simply regarded as "many." Even in modern times, there are tribes in the Amazon region and the jungles of Central America that only count as high as two or three. They've never felt the need to count any higher. But those tribes are the exception. In most of the ancient world, people began to recognize larger quantities, and they added new words for them to their languages.

Other developments followed. At some point, people began to think of numbers as abstract concepts. Without that, the process of counting is different for everything being counted. "Two rocks," for example, is considered different from "two cows." But gradually people began to recognize "two" not just as a property that applied to rocks or cows but as a concept in its own right, a number that could be understood even when it wasn't applied to anything. No one knows how long it took for people to make this mental leap, but most estimates are in the thousands of years. The ancient world changed much more slowly than the world does today.

As the understanding of numbers grew, so did the ways of communicating them. After all, it doesn't do much good to know the number of animals you've seen on your hunt, for example, if you can't tell the others in your tribe about it. The first people to try conveying numbers to others probably counted one by one on their fingers, just as we still do. But that method is limited to the count of ten. Representing larger numbers required more complex ways of arranging hands and fingers. Some of these traditional systems are still used in parts of the world, and new ones have been invented for American Sign Language and other visual languages.

Other societies counted with other parts of their bodies to reach beyond ten. Including their toes got them to twenty, but they didn't stop there. As recently as the last century, anthropologists studying villages in New Guinea and the other islands north of Australia found that the counting system stretched well beyond the digits.



People using this method simply pointed to the appropriate body part to communicate a number to someone else. It's simpler than remembering a combination of gestures but not very useful for anyone wanting to do arithmetic.

Humanity reached a major turning point about ten thousand years ago, when people began growing their food and keeping domesticated livestock. Without the need for hunting and gathering, they settled down in small villages. By eight thousand years ago, some of those villages had grown to become the first towns and cities. Civilization had begun, and with it came challenges that required a lot more mathematics than was possible by counting on fingers, toes, or any other body parts. Fortunately, civilization brought something else, too: the first written languages. Writing enabled the first civilizations to work the first math problems and record the first solutions, not just for their own benefit but also for later generations, right up to our own.

We're going to focus on three of these early civilizations. They were not the only ones to develop mathematics, nor were their mathematics necessarily the most advanced, but they will give us a good look at the different ways in which people first worked with numbers.

Chapter 7

The Land of the Mathematical King

The world's first civilization—that is, the first place we know of where people built cities and organized themselves into a society—was in a region called Sumer, now in the southern part of Iraq. Today the region is also called Mesopotamia (Greek for "between rivers") because it lies between two major rivers, the Tigris and the Euphrates. It was an ideal place for civilization to begin, with an abundant water supply and rich soil.

Starting in about 3500 B.C.E., the Sumerians established a number of city-states, largely isolated and independent of one another, each ruled by a local king. But by 3000 B.C.E., these kings were challenging each other for land and power. The Sumerian landscape was (and still is) wide-open and flat, with few features that a city-state could use as a natural defense. Wars and invasions were a constant threat for more than two thousand years, as empires rose and fell and royal dynasties came and went. In the first half of the eighteenth century B.C.E., King Hammurabi of Babylon conquered all of Sumer, effectively ending the region's independence.



The last great Sumerian empire is known to historians as "Ur III" because it was the third dynasty established in the city of Ur, located near the modern-day city of Nasiriyah. For more than two centuries, Sumer had been ruled by empires from the north, most notably the city of Akkad, which was near present-day Baghdad. But by 2112 B.C.E., those empires had fallen, and the city-states were again battling one another for power. A military general named Ur-Nammu emerged the winner, founding a kingdom that would continue through four successors for more than a century.

By Ur-Nammu's time, the people of Mesopotamia were already using mathematics for business, land measurement, and collecting taxes. But instead of paper or parchment, they recorded their work on small slabs of clay, making marks with a wedge-shaped reed stylus in the world's first form of writing, *cuneiform*.



The world's first "tablet." Many such tablets were about the same size as a modern smartphone.

The scribes who wrote and kept these tablets learned their trade in a school called an *edubba* (or, in the Akkadian language, *bit-tuppi*, which literally means "tablet house"). Scribes had an important role in Sumerian society, and not everyone could join their ranks. Most of the students were boys, although archaeologists have found evidence that there were also girls. Classroom ruins that have been discovered feature rows of low brick benches where the students sat and worked with practice tablets. Some of them also included low water troughs where fresh tablets could be made. It took many years of study to become a scribe, learning to write in both Sumerian and Akkadian, and

especially learning how to solve the math problems that the scribes could expect to face in their professional lives.

Ur-Nammu ruled for eighteen years before he was killed in battle. In addition to his military conquests, he established the first known written legal code—three centuries before the better-known Code of Hammurabi. The dynasty reached its peak, however, during the forty-eight year rule of his son, Shulgi. The decades from 2094 to 2046 B.C.E. saw the empire grow and prosper, stretching its influence all the way to the Mediterranean coast. At the same time, its culture reached new heights in literature and music, and its educational system flourished, thanks to two large *edubbas* that Shulgi established. Shulgi was in many ways the ideal king—or at least, that was the way he saw it.



Shulgi of Ur. This figure was buried in the foundation of a temple and shows him carrying building materials for the temple.

Monarchs throughout history have been the subject of stories, poems, songs, and other works of art. The best-known Sumerian example, the *Epic of Gilgamesh*, was written sometime around Shulgi's rein and is probably based on an earlier king who ruled from the city of Uruk. But the poetry written about the kings of Shulgi's time include a feature that might seem strange to us today—something we might see as beneath a king's dignity. In these poems, the king himself joins in his own adulation, boasting in long first-person passages about his skills and accomplishments. We'll never know whether any of the kings actually wrote these passages themselves or had "ghost writers" write them. Shulgi seems like someone who might have tried it. In the epic *Shulgi the*

Ideal King, he boasts not just about his skills in fighting, hunting, music, and fortune-telling but also about his mathematical ability, claiming:

> *As a youth, I studied the scribal art in the edubba, from the tablets of Sumer and Akkad,*
>
> *Of the nobility, no one was able to write a tablet like me,*
>
> *In the place where the people attend to learn the scribal art,*
>
> *Adding, subtracting, counting, and accounting—I completed all of their courses.*

As king, Shulgi established laws and policies that made mathematical skills important. He expanded the government bureaucracy and established a new bookkeeping and accounting system, keeping track of land ownership, food supplies, and many other parts of Sumerian life. He also changed the Sumerian calendar and established new standards for weighing and measuring. And he built new shrines and temples throughout the kingdom, most notably finishing the large ziggurat in Ur that his father had begun.



Despite the great heights of Shulgi's rule, Sumer would not continue as an independent kingdom for much longer. In 2004 B.C.E., the kingdom fell to the Elamites and was then ruled by a series of empires until modern times. In contrast, Sumerian mathematics continued for centuries, adopted by those empires and developed by their scholars. In fact, *you* have used some Sumerian mathematics in your daily life—without ever realizing it. But we'll find out more about that later.